

1.4 Handwrite

Welcoming Environment: Actively listen to others and encourage everyone to participate! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter. Discuss and keep track of any questions your group has. Ask me questions during group work time as well as when I bring us back together. Try to help each other solidify and review the language of linear algebra, algebra, visualizations and intuition from this section, including those related to:

- algebra of matrix vector equation $A\vec{x} = \vec{b}$:
 - multiply a matrix and a column vector by linear combinations of the columns of A using weights from \vec{x}
 - span of the columns of A = set of all linear combinations of the columns of A
 - matrix vector equation \rightarrow vector equation \rightarrow augmented matrix
 - equations generic vectors \vec{b} must satisfy to be in the span (Example 3)
 - dot products of rows of A with \vec{x}
- geometry of solutions of matrix vector equation $A\vec{x} = \vec{b}$ and geometry of spaces of subsets of \mathbb{R}^3 spanned by the column vectors of A (Figure 1)
- Theorem 4: relationship of consistency of $A\vec{x} = \vec{b}$ to always being a linear combination to spanning the entire \mathbb{R}^m , where m is the number of rows, to having a pivot position in every row of A .
- identity matrix I

Take out your notes from the activities due today as well as the fill-in guide. Use them and each other to respond to the following by handwriting in the language of our class. Use only what we have covered so far in our readings, videos and quizzes.

1. **Building Community:** What are the preferred first names of those sitting near you? If you weren't able to be there, give reference to anyone you had help from or write N/A otherwise.

2. Let $A = \begin{bmatrix} 3 & -1 \\ -9 & 3 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ and consider the matrix equation $\begin{bmatrix} 3 & -1 \\ -9 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$.

- Show that the equation $A\vec{x} = \vec{b}$ does not have a solution for all possible \vec{b} by applying strict Gaussian to the corresponding augmented matrix—show replacement and find an inconsistency.
- Applying a), write an equation using b_1 and b_2 that \vec{b} in the span satisfy for consistency (hint: turn the bottom right spot into a consistency equation).
- Show that the coordinates of each of A 's column vectors satisfy the equation in b), like $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -9 \end{bmatrix}$.
- Sketch and label the two column vectors of A using standard mathematical axes.
- What is the geometry of the span of the columns of A ?

3. Suppose A is a 4×3 matrix and \vec{b}_1 is one vector in \mathbb{R}^4 with the property that $A\vec{x} = \vec{b}_1$ has a unique solution.
- a) What does the 4×3 tell us about the number of rows and number of columns of A ?
 - b) Review the idea of a pivot and fill-in: A pivot is the first non-zero number across a _____ in the Gaussian or Gauss-Jordan form of a matrix.
 - c) What does $A\vec{x} = \vec{b}_1$ having a unique solution tell us about the pivot columns of the coefficient matrix—could a column of the coefficient matrix be missing a pivot after $[A\vec{b}_1]$ is reduced?
 - d) How many pivots does A have?
 - e) What will the reduced row echelon form (Gauss-Jordan with only 1s and 0s) of A , the coefficient matrix, be?
 - f) Do the columns of A span all of \mathbb{R}^4 ? Justify your response using theorem 4, which I summarized for you on the first page of this handwrite.

Next, as time allows before I bring us back together, work on the additional activities including any pollev activities and respond in your notes rather than here.

Help each other and PDF responses to ASULearn: If you are finished with the handwrite and additional activities before I bring us back together, first ensure that your entire group is finished too, and if not, help each other. Then submit your handwrite, continue reviewing and solidifying or discuss upcoming class work.

Collate your handwritten responses, preferably on this handout, into one full size multipage PDF for submission in the ASULearn assignment. I recommend you turn it in sometime today, but you have until the morning before the next class.