You can preview this quiz, but if this were a real attempt, you would be blocked because:

This quiz is not currently available

Question **1**

Not complete

Points out of 5.00

For $A = \begin{bmatrix} 1 & 2 \\ -3 & 1 \\ 1 & 6 \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ compute the product using the two methods below.

a) multiply a matrix and a column vector $A\vec{x}$ via the linear combination method

b) multiply a matrix and a column vector $A\vec{x}$ via the row-vector rule

Part a) The linear combination method:

○ is not defined because there is no constant for the last row $-2\begin{bmatrix} 1 & 2 \end{bmatrix} + 3\begin{bmatrix} -3 & 1 \end{bmatrix} + ?\begin{bmatrix} 1 & 6 \end{bmatrix}$

○ is $-2\begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$

○ other

Part b) the row-vector rule:

○ is not defined because the dot product of $\begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$ does not exist

○ is $\begin{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 3 \end{bmatrix} \\ \begin{bmatrix} -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 3 \end{bmatrix} \\ \begin{bmatrix} 1 & 6 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{bmatrix}$

○ other

Part c) Compute the result, if it exists. If any or all entries of the following vector do not exist, type DNE. Otherwise simplify and type in a number in the relevant spots, with no extra spaces or other characters.

[ ]

[ ]

[ ]

Check

Question **2**

Not complete

Points out of 2.00

Write the following system as a vector equation (linear combination of vectors) and then as a matrix vector equation.

$3x_1 + x_2 - 5x_3 = 9$

$x_2 + 4x_3 = 0$

Part a) For the vector equation (linear combination of vectors), where do the variables go?

- ○ they are in a row vector $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$
- ○ they are in a column vector $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$
- ○ they are scalar weights of the linear combination $x_1 \begin{bmatrix} a \\ b \end{bmatrix} + x_2 \begin{bmatrix} c \\ d \end{bmatrix} + x_3 \begin{bmatrix} e \\ f \end{bmatrix}$
- ○ other

Part b) As a matrix vector equation $A\vec{x} = \vec{b}$?

- ○ is not defined
- ○ $\begin{bmatrix} 3 & 0 \\ 1 & 1 \\ -5 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$
- ○ other

Check

Question **3**

Not complete

Points out of 19.00

Given the matrix equation $\begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$, multiply the left side and write a system of equations. Next write the augmented matrix for the system.

What is the corresponding augmented matrix---notice that I've filled in the equal column for you?

			b_1
			b_2
			b_3

What is the first step of strict Gaussian (use replacement but not scaling) and for the purpose of ASULearn, don't swap rows either, although that is allowed in strict Gaussian.

$r_3' =$

$r_1 + r_3$

What is the matrix after applying this next replacement?

-1	2	3	b_1
0	1	2	b_2
			$b_1 + b_3$

What is the next step in strict Gaussian?

$r_3' =$

$r_2 + r_3$

What is the matrix after applying this one replacement?

-1	2	3	b_1
0	1	2	b_2
			$b_1 - 2b_2 + b_3$

The matrix is now in row echelon form.

What is an equation for vectors $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ in the span of the columns of the original coefficient matrix, if any?

- ○ there is no such equation
- ○ there is an equation but more work needs to be done
- ○ the intersection of $b_1 + b_3 = 0$ and $b_1 - 2b_2 + b_3 = 0$
- ○ $b_1 - 2b_2 + b_3 = 0$
- ○ other

What is the span of the columns of the original coefficient matrix?

- ○ nothing
- ○ $\vec{0}$
- ○ a line through the origin
- ○ a plane through the origin
- ○ all of $\mathbb{R}^3$

Check

Question **4**

Not complete

Points out of 2.00

Given the matrix equation $\begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$, open
https://www.geogebra.org/m/aamthwjp

and drag the sliders c_1, c_2, c_3 to actively engage with the generic vector $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ in the equal column that gives consistency to the matrix equation. After using all three sliders, can you turn the consistent $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ so that all of them are in the same plane, i.e. a "head on" view?

○ yes, the span of the columns of the coefficient matrix is visualized as an infinite plane

○ no, the span is visualized as all of 3-space, an infinite volume

Given this second matrix equation $\begin{bmatrix} -1 & 2 & -3 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$, where I changed the 3 to a -3, open
https://www.geogebra.org/m/cykxpshw

and drag the sliders c_1, c_2, c_3 to actively engage with the generic vector $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ in the equal column that gives consistency to this second matrix equation. After using all three sliders, can you turn the consistent $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ so that all of them are in the same plane, i.e. a "head on" view?

○ yes, the span of the columns of the coefficient matrix is visualized as an infinite plane

○ no, the span is visualized as all of 3-space, an infinite volume

Check

Question **5**

Not complete

Points out of 4.00

Given $\begin{bmatrix}5\\0\\0\end{bmatrix}$, $\begin{bmatrix}7\\2\\-6\end{bmatrix}$, $\begin{bmatrix}9\\4\\-8\end{bmatrix}$, what vector equation would directly solve for whether the 3 vectors span all of $\mathbb{R}^3$ or not?

○ $c_1\begin{bmatrix}5\\0\\0\end{bmatrix}+c_2\begin{bmatrix}7\\2\\-6\end{bmatrix}=\begin{bmatrix}9\\4\\-8\end{bmatrix}$ is consistent

○ $c_1\begin{bmatrix}5\\0\\0\end{bmatrix}+c_2\begin{bmatrix}7\\2\\-6\end{bmatrix}+c_3\begin{bmatrix}9\\4\\-8\end{bmatrix}=\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$ is consistent

○ $c_1\begin{bmatrix}5\\0\\0\end{bmatrix}+c_2\begin{bmatrix}7\\2\\-6\end{bmatrix}+c_3\begin{bmatrix}9\\4\\-8\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$ has only the trivial solution

○ more than one of the above

Which system is the matrix vector equation corresponding to the vector equation that would directly solve for whether the 3 vectors span all of $\mathbb{R}^3$ or not?

○ is not defined

○ $\begin{bmatrix}5&0&0\\7&2&-6\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}9\\4\\-8\end{bmatrix}$

○ $\begin{bmatrix}5&0&0\\7&2&-6\\9&4&-8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$

○ $\begin{bmatrix}5&7&9\\0&2&4\\0&-6&-8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$

○ $\begin{bmatrix}5&0&0\\7&2&-6\\9&4&-8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

○ $\begin{bmatrix}5&7&9\\0&2&4\\0&-6&-8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

○ more than one of the above

Write the augmented matrix for the system. Notice that this reducing to row echelon form using strict Gaussian will only require one step---just one replacement!

What is this step? $r_3' =$

[ ]

$r_2 + r_3$

How many solutions does the system have?

- ○ 0 always
- ○ 0 for some $\vec{b}$
- ○ 1
- ○ ∞

So, do the original 3 vectors span all of $\mathbb{R}^3$?

- ○ yes
- ○ no

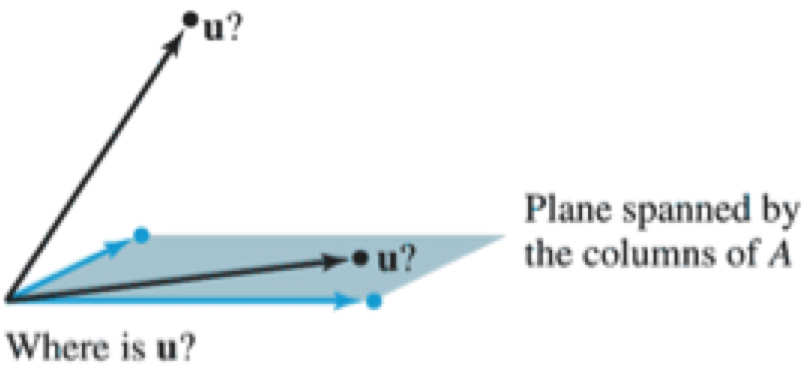
Check

Question **6**

Not complete

Points out of 2.00

Let $\vec{u} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$ and $A = \begin{bmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{bmatrix}$. Is $\vec{u}$ in the plane in $\mathbb{R}^3$ spanned by the columns of A (see the figure)?



Part a) Set up the relevant augmented matrix for the matrix vector system $A\vec{x} = \vec{u}$ and reduce it using Gaussian. How many solutions does the system have?

○0

○1

○∞

Part b) Is $\vec{u}$ in the plane spanned by the columns of *A*?

○yes

○no because the first coordinate of $\vec{u}$ has a 0 but the columns of A don't

○no, other reason

Part c) Inside of Maple, you should be able to paste and execute the following code (if copy and paste don't work for you, you can type it in directly).

```
with(LinearAlgebra): with(plots):
plot1b:=spacecurve({[3*t, -2*t, 1*t, t = 0 .. 1]}, color = "Niagara Azure", thickness = 3):
plot2b:=spacecurve({[-5*t,6*t,1*t,t = 0 .. 1]}, color = "Spring Orange", thickness = 4, linestyle=SpaceDash):
plot3b:=spacecurve({[0*t, 4*t, 4*t, t = 0 .. 1]},color="Grey",thickness = 5, linestyle=SpaceDot):
display(plot1b,plot2b,plot3b);
```

Can we turn the plot "head on" so that all three vectors are revealed to lie in the same plane with none of them sticking out of the "head on" view?

○yes

○no

Check

Question **7**

Not complete

Points out of 2.00

Let A be a 3×2 matrix. Explain why the equation $A\vec{x} = \vec{b}$ cannot be consistent for all $\vec{b}$ in $\mathbb{R}^3$. Then examine the case of an arbitrary A with more rows than columns.

3x2 Matrix To explain why the equation $A\vec{x} = \vec{b}$ cannot be consistent for all $\vec{b}$ in $\mathbb{R}^3$ when A is 3×2, which of the following are valid arguments?

○a) $\vec{b}$ in $\mathbb{R}^3$ is the wrong size vector for a 3×2 matrix

○b) There are only 2 columns so A can have at most 2 pivots, which is not enough to fill all three rows, so by Theorem 4 part d) is false and hence part a) is false.

○c)There will be some $\vec{b}$ vectors that in Gauss Jordan reduction (reduced row echelon form) on the augmented matrix for the system will yield [0 0 nonzero], because A has too many rows to have a pivot in each one of them.

○part a) and part b) are both valid arguments.

○part a) and part c) are both valid arguments

○part b) and part c) are both valid arguments.

Larger Matrix Next examine the case of an arbitrary A with more rows m than columns n. Can $A\vec{x} = \vec{b}$ be consistent for all $\vec{b}$ in $\mathbb{R}^m$?

○No. We can apply similar reasoning to above.

○Yes. higher dimensions gives us consistency.

Check

Question **8**

Not complete

Points out of 1.00

Is the statement "A vector $\vec{b}$ is a linear combination of the columns of a matrix A if and only if the equation $A\vec{x} = \vec{b}$ has at least one solution" true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

○True and I found a phrase and page number from the text

○False and I can provide a counterexample

○other

Check

Question **9**

Not complete

Points out of 1.00

Is the statement "If A is an $m \times n$ matrix and if the equation $A\vec{x} = \vec{b}$ is inconsistent for some $\vec{b}$ in $\mathbb{R}^m$, then A cannot have a pivot position in every row" true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

- True and I found a phrase and page number from the text
- False and I can provide a counterexample
- other

Check

Question **10**

Not complete

Points out of 1.00

To solidify and prepare for upcoming work, review and contemplate your knowledge and any questions that remain as related to definitions, concepts, computations, and examples from 1.4, including

- algebra of matrix vector equation $A\vec{x} = \vec{b}$:
  - multiply a matrix and a column vector by linear combinations of the columns of *A* using weights from $\vec{x}$
  - span of the columns of *A* = set of all linear combinations of the columns of *A*
  - matrix vector equation $\longrightarrow$ vector equation $\longrightarrow$ augmented matrix
  - equations generic vectors $\vec{b}$ must satisfy to be in the span (Example 3)
  - dot products of rows of *A* with $\vec{x}$,
- geometry of solutions of matrix vector equation $A\vec{x} = \vec{b}$: spaces of subsets of R^3 spanned by the column vectors of A, geometry of such spaces (Figure 1)
- Theorem 4: relationship of consistency of $A\vec{x} = \vec{b}$ to always being a linear combination to spanning the entire R^m, where *m* is the number of rows, to having a pivot position in every row of *A*.
- identity matrix *I*

Consider also 1.3, including

- algebra of vectors: coordinates, addition of vectors, scalar multiplication of vectors, properties like associativity under addition (property ii on p. 29), a linear combination with weights, zero vector, span of a set of vectors=all the linear combinations, is a vector in the span?, vector equation $\longrightarrow$ augmented matrix
- geometry of vectors in 2D and 3D: directed segment, parallelogram for addition, on same line for scalar multiplication of vectors, origin=zero vector, a linear combination geometrically in the plane or 3D, span=all the linear combinations geometrically in the plane or 3D, spaces of subsets of R^n spanned by vectors

and 1.2, including

- matrix of a linear system: row echelon form (Gaussian), reduced row echelon form (Gauss-Jordan)
- pivots: pivot position of a matrix, pivot column of a matrix
- row reduction algorithm we will most commonly use: elimination by forward phase and back substitution to row echelon form
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions with free variables and parametric solutions

and 1.1, including:

- algebra of linear equations: coefficients and variables
- geometry of linear equations in 2D and 3D: lines and planes
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions
- matrix of a linear system: coefficient matrix, augmented matrix, triangular form
- row equivalent systems
- algorithm for solving a linear system using elementary row operations of replacement, interchange, and scaling

When you have finished reviewing and reflecting, select one of the following (both receive full credit)

○ I currently have no questions

○ I will continue solidifying and understand that help is available in Dr. Sarah's more extensive feedback that follows below each question after I finish and open back up an entire practice quiz (this is more extensive than the hints that I can access during the open quiz), in Dr. Sarah's glossary/Wiki which is embedded into ASULearn from the linked terms, in Dr. Sarah's office hours and forum, and in Math Lab and Tutoring

Check