You can preview this quiz, but if this were a real attempt, you would be blocked because:

This quiz is not currently available

Question **1**

Not complete

Points out of 1.00

The matrix equation $\begin{bmatrix}1 & 1 & -3\\ 2 & 4 & -4\end{bmatrix}\begin{bmatrix}c_1\\ c_2\\ c_3\end{bmatrix} = \begin{bmatrix}10\\ 30\end{bmatrix}$ has infinite solutions because using strict Gaussian (replacement but no scaling to eliminate items below pivots), we see $\begin{bmatrix}1 & 1 & -3 & 10\\ 2 & 4 & -4 & 30\end{bmatrix} \xrightarrow{r_2'=-2r_1+r_2} \begin{bmatrix}1 & 1 & -3 & 10\\ 0 & 2 & 2 & 10\end{bmatrix}$,
which has z free as it has no pivot. Then by row 2, $2y = 10 - 2z$ so $y = 5 - z$. Substituting both of these into row 1, we have $x = 10 - y + 3z = 10 - (5 - z) + 3z = 5 + 4z$. Thus the parameterization is $\begin{bmatrix}5+4z\\ 5-z\\ z\end{bmatrix} = z\begin{bmatrix}4\\ -1\\ 1\end{bmatrix} + \begin{bmatrix}5\\ 5\\ 0\end{bmatrix}$

Open
https://www.geogebra.org/m/yv7fbv6u
and drag the x_3 slider to visualize how all the vectors end on a line. You can also turn the graph in 3-space. Work to solidify the visualization.

In the visualization, is line the vectors end on parallel to $v = \begin{bmatrix}5\\ 5\\ 0\end{bmatrix}$ or $w = \begin{bmatrix}4\\ -1\\ 1\end{bmatrix}$ or neither?

- ○ v
- ○ w
- ○ neither

Check

Question **2**

Not complete

Points out of 1.00

What's a free variable and how does it relate to a parameterization?

- ○ The free variables are the ones in row echelon form without pivots and they are the parameters in the parameterization, which also give us the geometry.
- ○ The free variables are unrelated to parameterization.
- ○ The free variables are the inconsistent variables.

Check

Question **3**

Not complete

Points out of 3.00

How does the number of free variables in a parameterization relate to the geometry?

1 free variable in a parameterization of solutions to a linear system is a

- ○ line
- ○ plane
- ○ infinite volume
- ○ other

2 free variables in a parameterization of solutions to a linear system is a

- ○ line
- ○ plane
- ○ infinite volume
- ○ other

3 free variables in a parameterization of solutions to a linear system is a

- ○ line
- ○ plane
- ○ infinite volume
- ○ other

Check

Question **4**

Not complete

Points out of 26.00

Given the matrix equation $\begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, multiply the left side and write a system of equations. Next write the augmented matrix for the system.

What is the corresponding augmented matrix?

☐	☐	☐	☐
☐	☐	☐	☐
☐	☐	☐	☐

What is the first step of strict Gaussian (use replacement but not scaling) and for the purpose of ASULearn, don't swap rows either, although that is allowed in strict Gaussian.

$r_3' =$

☐

$r_1 + r_3$

In your notes, write the matrix that you obtain after this one replacement. What is the next step in strict Gaussian?

$r_3' =$

☐

$r_2 + r_3$

What is the augmented matrix after applying these two replacements, i.e. the row echelon form?

☐	☐	☐	☐
☐	☐	☐	☐
☐	☐	☐	☐

Which variables have pivots?

- ○ only x_1
- ○ only x_2
- ○ only x_3
- ○ x_1 and x_2
- ○ x_1 and x_3

○ x_2 and x_3

○ x_1, x_2, and x_3

Notice that the augmented matrix is consistent since there is no row of [0 0 0 nonzero] in row echelon form. Any variables without pivots in a consistent system in row echelon form are free. Which variables are free?

○ x_1

○ only x_2

○ only x_3

○ x_1 and x_2

○ x_1 and x_3

○ x_2 and x_3

○ x_1, x_2, and x_3

Keep the free variables as free parameters. We can set them as time parameters, for instance, or keep them as they are listed. So for example, if there is 1 variable missing pivots in the reduced matrix, it can get a variable like *t* and if 2 were missing we could have an *s* and *t*. Then solve for any variables with pivots in terms of the parameters using the equations corresponding to any nonzero rows in row echelon form.

What is x_2?

○ it stays as x_2 or equivalently a parameter t since it is free

○ x_1

○ x_3

○ $-2x_3$

○ other

What is x_1?

○ it stays as x_1 or equivalently a parameter t since it is free

○ x_2

○ x_3

○ $-x_3$

○ other

Next write the solutions in vector form as a column vector. The free variables stay free while the pivot variables are written in terms of the parameters. Using this method of parameterization, which of the following represents the solutions at this step?

○ $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

○ $\begin{bmatrix} -x_3 \\ 2x_3 \\ 3x_3 \end{bmatrix}$

○ $\begin{bmatrix} -x_3 \\ -2x_3 \\ x_3 \end{bmatrix}$

○ other

The last step of parameterization is to factor out any free variables. Which of the following describes the solutions geometrically and in parametric vector form?

- ○ no solutions as there are no concurrent intersections
- ○ a line of solutions $x_3 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
- ○ a line of solutions $x_3 \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$
- ○ a plane of solutions $x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$
- ○ an infinite volume of solutions $x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$
- ○ other

Check

Question **5**

Not complete

Points out of 1.00

Suppose that the augmented matrix for a system reduces to $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Which variables have pivots?

- ○only x_1
- ○only x_2
- ○only x_3
- ○x_1 and x_2
- ○x_1 and x_3
- ○x_2 and x_3
- ○x_1, x_2, and x_3

Notice that the augmented matrix is consistent since there is no row of [0 0 0 nonzero] in row echelon form. Any variables without pivots in a consistent system in row echelon form are free. Which variables are free?

- ○x_1
- ○only x_2
- ○only x_3
- ○x_1 and x_2
- ○x_1 and x_3
- ○x_2 and x_3
- ○x_1, x_2, and x_3

Keep the free variables as free parameters. We can set them as time parameters, for instance, or keep them as they are listed. So for example, if there is 1 variable missing pivots in the reduced matrix, it can get a variable like *t* and if 2 were missing we could have an *s* and *t*. Then solve for any variables with pivots in terms of the parameters using the equations corresponding to any nonzero rows in row echelon form. What is x_1?

- ○it stays as x_1 or equivalently a parameter t since it is free
- ○x_3
- ○$-2x_2$
- ○$-2x_2 - 3x_3$
- ○other

Next write the solutions in vector form as a column vector. The free variables stay free while the pivot variables are written in terms of the parameters. Using this method of parameterization, which of the following represents the solutions at this step?

- ○$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$
- ○$\begin{bmatrix} x_3 \\ 2x_3 \\ 3x_3 \end{bmatrix}$

○$\begin{bmatrix} -2x_2 - 3x_3 \\ x_2 \\ x_3 \end{bmatrix}$

The last step of parameterization is to factor out any free variables. Which of the following describes the solutions geometrically and in parametric vector form?

○no solutions as there are no concurrent intersections

○a line of solutions $x_3 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

○a plane of solutions $x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$

○an infinite volume of solutions $x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

○other

Open
https://www.geogebra.org/m/n4xwnuuk
If the sliders aren't already both playing, start them. After the two free sliders have had the chance to go around, turn the graph to look for a "head on" view of a line or plane the vectors are in, if that is possible. What does the visualization show?

○a "head on" view is possible

○vectors are always sticking out, no matter how the graph is turned so there is no "head on" view

Check

Question **6**

Not complete

Points out of 2.00

Determine if this homogeneous system has a nontrivial solution. Try to use as few row operations as possible.

$$-3x_1 + 4x_2 - 8x_3 = 0$$
$$-2x_1 + 5x_2 + 4x_3 = 0$$

Part a) Does the system have a nontrivial solution?

- No. The system has only a trivial solution
- No. The system has no solutions
- Yes. The system has a nontrivial solution
- It is impossible to determine

Part b) The instructions say: "Try to use as few row operations as possible." What is the smallest number of row operations we can use here?

- 0
- 1
- other

Check

Question **7**

Not complete

Points out of 1.00

To write the solutions of $A\vec{x} = \vec{0}$ in parametric vector form, where A is row equivalent to the given matrix $\begin{bmatrix} 3 & -6 & 6 \\ -2 & 4 & -2 \end{bmatrix}$.

First write down the augmented matrix in your notes.

Next apply Gaussian to your augmented matrix and reduce to row echelon form.

Which variables have pivots?

- ○ only x_1
- ○ only x_2
- ○ only x_3
- ○ x_1 and x_2
- ○ x_1 and x_3
- ○ x_2 and x_3
- ○ x_1, x_2, and x_3

Parameterize the solutions by writing any variables with pivots using the rows with their pivots, leaving any parameters as free. What is x_3?

- ○ it stays as x_3 or equivalently a parameter t since it is free
- ○ x_2
- ○ 0
- ○ other

Write the solutions in parameterized vector form and factor out any free variables. What are the solutions?

- ○ The system is inconsistent.
- ○ the point $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
- ○ the line $x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ or equivalently $t \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$
- ○ the line $x_3 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ or equivalently $t \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$
- ○ the plane $x_1 \begin{bmatrix} 3 \\ -6 \\ 6 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$
- ○ other

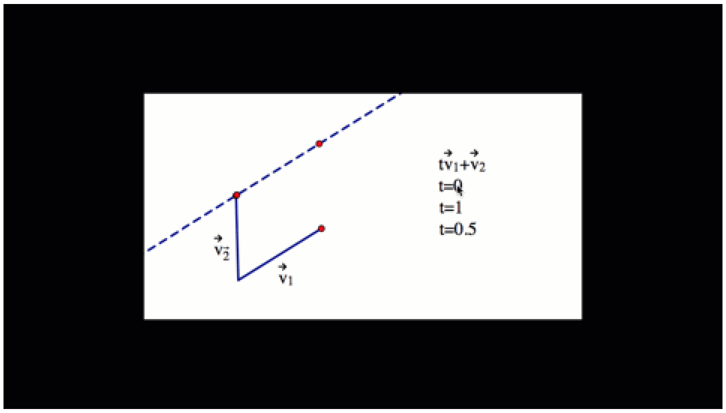
Check

Question **8**

Not complete

Points out of 1.00

What does this animated gif show?



○ vectors that end on the line parallel to $\vec{v}_1$ through the tip of $\vec{v}_2$ are formed as we vary the coefficient of $\vec{v}_1$ in $t\vec{v}_1 + \vec{v}_2$

○ the span of $\vec{v}_1$ and $\vec{v}_2$ is a plane

○ both

○ none of the above

Check

Question **9**

Not complete

Points out of 1.00

Find the parametric equation of the line through $\begin{bmatrix} -2 \\ 0 \end{bmatrix}$ parallel to $\begin{bmatrix} -5 \\ 3 \end{bmatrix}$

○ $t\begin{bmatrix} -2 \\ 0 \end{bmatrix} + \begin{bmatrix} -5 \\ 3 \end{bmatrix}$

○ $t\begin{bmatrix} -5 \\ 3 \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

○ other

Check

Question **10**

Not complete

Points out of 1.00

Suppose the solution set of a certain system of linear equations can be described as

$x_1 = 5 + 4x_3$

$x_2 = -2 - 7x_3$

with x_3 free.

Use vectors to describe this set as a line in $\mathbb{R}^3$

○This is a line through the tip of $\begin{bmatrix} 4 \\ -7 \\ 1 \end{bmatrix}$ and parallel to $\begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix}$

○This is a line through the tip of $\begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix}$ and parallel to $\begin{bmatrix} 4 \\ -7 \\ 1 \end{bmatrix}$

○other

Check

Question **11**

Not complete

Points out of 1.00

Is the statement "A homogeneous equation (homogeneous system) always consistent" true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

○True and I found a phrase and page number from the text

○False and I can provide a counterexample

○other

Check

Question **12**

Not complete

Points out of 1.00

Is the statement "The equation $\vec{x} = \vec{p} + t\vec{v}$ describes a line through $\vec{v}$ parallel to $\vec{p}$" true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

○True and I found a phrase and page number from the text

○False and I can provide a counterexample

○other

Check

Question **13**

Not complete

Points out of 1.00

To solidify and prepare for upcoming work, review and contemplate your knowledge and any questions that remain as related to definitions, concepts, computations, and examples from 1.5, including

- algebra of homogeneous systems: $A\vec{x} = \vec{0}$
- algebraic solutions of homogenous systems always include the trivial solution=$\vec{0}$. nontrivial solutions, if any exist, are parameterized in parametric vector form using free variables to express those as well as the variables with pivots and then decomposed algebraically to showcase the algebra and geometry giving $t\vec{v}$ or $s\vec{u} + t\vec{v}$ or similar, where each free variable is attached to a vector.
- geometry of solutions of homogeneous systems are geometric spaces through the origin like lines, planes, or hyperplanes
- algebra of nonhomogeneous systens: $A\vec{x} = \vec{b}$
- solutions of nonhomogeneous systems in parametric vector form can be decomposed algebraically to showcase the algebra and geometry like $\vec{p} + t\vec{v}$, vectors ending on the line parallel to $\vec{v}$ or $\vec{p} + s\vec{u} + t\vec{v}$, vectors ending on the plane parallel to the one spanned by $\vec{u}, \vec{v}$...
- geometry of solutions of nonhomogeneous systems are geometric spaces translated away from the origin via adding $\vec{p}$

Consider also 1.4, including

- algebra of matrix vector equation $A\vec{x} = \vec{b}$:
  - multiply a matrix and a column vector by linear combinations of the columns of *A* using weights from $\vec{x}$
  - span of the columns of *A* = set of all linear combinations of the columns of *A*
  - matrix vector equation $\longrightarrow$ vector equation $\longrightarrow$ augmented matrix
  - equations generic vectors $\vec{b}$ must satisfy to be in the span (Example 3)
  - dot products of rows of *A* with $\vec{x}$,
- geometry of solutions of matrix vector equation $A\vec{x} = \vec{b}$: spaces of subsets of R^3 spanned by the column vectors of A, geometry of such spaces (Figure 1)
- Theorem 4: relationship of consistency of $A\vec{x} = \vec{b}$ to always being a linear combination to spanning the entire R^m, where *m* is the number of rows, to having a pivot position in every row of *A*.
- identity matrix *I*

and 1.3, including

- algebra of vectors: coordinates, addition of vectors, scalar multiplication of vectors, properties like associativity under addition (property ii on p. 29), a linear combination with weights, zero vector, span of a set of vectors=all the linear combinations, is a vector in the span?, vector equation $\longrightarrow$ augmented matrix
- geometry of vectors in 2D and 3D: directed segment, parallelogram for addition, on same line for scalar multiplication of vectors, origin=zero vector, a linear combination geometrically in the plane or 3D, span=all the linear combinations geometrically in the plane or 3D, spaces of subsets of R^n spanned by vectors

and 1.2, including

- matrix of a linear system: row echelon form (Gaussian), reduced row echelon form (Gauss-Jordan)
- pivots: pivot position of a matrix, pivot column of a matrix
- row reduction algorithm we will most commonly use: elimination by forward phase and back substitution to row echelon form
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions with free variables and parametric solutions

and 1.1, including:

- algebra of linear equations: coefficients and variables
- geometry of linear equations in 2D and 3D: lines and planes
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions
- matrix of a linear system: coefficient matrix, augmented matrix, triangular form
- row equivalent systems
- algorithm for solving a linear system using elementary row operations of replacement, interchange, and scaling

When you have finished reviewing and reflecting, select one of the following (both receive full credit)

○ I currently have no questions

○ I will continue solidifying and understand that help is available in Dr. Sarah's more extensive feedback that follows below each question after I finish and open back up an entire practice quiz (this is more extensive than the hints that I can access during the open quiz), in Dr. Sarah's glossary/Wiki which is embedded into ASULearn from the linked terms, in Dr. Sarah's office hours and forum, and in Math Lab and Tutoring

Check