You can preview this quiz, but if this were a real attempt, you would be blocked because:

This quiz is not currently available

Question **1**

Not complete

Points out of 2.00

Determine if the vectors are linearly independent and justify your answer.

$\begin{bmatrix} 2 \\ -3 \end{bmatrix}, \begin{bmatrix} -4 \\ 6 \end{bmatrix}$

Examine the following arguments for their validity:

1) There are only two vectors, so it suffices to check if one is redundant by being on the same line through the origin as the other.

2) We can inspect the vectors to solve for weights so that $c_1 \begin{bmatrix} 2 \\ -3 \end{bmatrix} + c_2 \begin{bmatrix} -4 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ or whether we have only the trivial solution.

3) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is not one of the vectors, so they can't be linearly independent.

Which of the following are valid arguments here?

○1) and 2)

○1) and 3)

○2) and 3)

○1), 2) and 3)

○other

Are the vectors linearly independent?

○yes

○no

Check

Question **2**

Not complete

Points out of 15.00

Determine if the vectors are linearly independent and justify your answer.

$$\begin{bmatrix}5\\0\\0\end{bmatrix},\begin{bmatrix}7\\2\\-6\end{bmatrix},\begin{bmatrix}9\\4\\-8\end{bmatrix}$$

What vector equation would solve for the definition of linear independent vectors?

- ○ $c_1\begin{bmatrix}5\\0\\0\end{bmatrix}+c_2\begin{bmatrix}7\\2\\-6\end{bmatrix}=\begin{bmatrix}9\\4\\-8\end{bmatrix}$ is consistent
- ○ $c_1\begin{bmatrix}5\\0\\0\end{bmatrix}+c_2\begin{bmatrix}7\\2\\-6\end{bmatrix}+c_3\begin{bmatrix}9\\4\\-8\end{bmatrix}=\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$ is consistent
- ○ $c_1\begin{bmatrix}5\\0\\0\end{bmatrix}+c_2\begin{bmatrix}7\\2\\-6\end{bmatrix}+c_3\begin{bmatrix}9\\4\\-8\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$ has only the trivial solution

Set up the augmented matrix for the system, preserving the ordering I used above:

☐	☐	☐	☐
☐	☐	☐	☐
☐	☐	☐	☐

Next reduce to Gaussian form. How many solutions does the system have?

○0 ○1 ○∞

Are the vectors linearly independent?

○yes ○no

Check

Question **3**

Not complete

Points out of 2.00

Which is true?

- ○ span is all the linear combinations
- ○ span is all the linearly independent vectors
- ○ both of the above
- ○ none of the above

Which is true?

- ○ we can span a space without being linearly independent
- ○ we can be linearly independent in a space without spanning the entire space
- ○ both of the above
- ○ none of the above

Check

Question **4**

Not complete

Points out of 3.00

Is $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ linearly independent?

- yes
- no
- sometimes

Compare set 1 $\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \right\}$ to set 2 $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \right\}$

- set 1 is linearly independent but set 2 is not
- set 2 is linearly independent but set 1 is not
- both sets are linearly independent
- neither set is linearly independent

What about the spans?

- The span of set 1 is the same plane as the span of set 2
- The spans are different
- other

Check

Question **5**

Not complete

Points out of 2.00

Prior 2240 student Nathan Bonfanti made some visualizations as a part of a project and shared them with me for use in future classes. What you'll see is linear combinations of 3 vectors in 3-space. Nathan has highlighted the individual vectors once they are scaled, which you'll see as 3 vectors, as well as their sum that gives the linear combination, shown as points for different weights in the linear combinations. The points are the endpoints of the linear combination vector to make them easier to see.

Open
https://www.geogebra.org/m/pwwnjqhj
If it isn't already playing, you can drag the vectors or click the circular arrow that starts it playing and watch the linear combinations form and solidify the visualization. Which is true?

- ○ We can turn the graph "head on" to show that the 3 vectors span a plane and hence are not linearly independent since only 2 vectors are needed to span a plan efficiently.
- ○ The 3 vectors span all of 3-space and since there are only 3 of them, they are linearly independent, because we couldn't span all of 3-space with less than 3 vectors.
- ○ other

Next, open
https://www.geogebra.org/m/je25vvmf
to watch the linear combinations of 3 new vectors form and solidify the visualization. Which is true?

- ○ We can turn the graph "head on" to show that the 3 new vectors span a plane and hence are not linearly independent since only 2 vectors are needed to span a plan efficiently.
- ○ The 3 new vectors span all of 3-space and since there are only 3 of them, they are linearly independent, because we couldn't span all of 3-space with less than 3 vectors.
- ○ other

Check

Question **6**

Not complete

Points out of 1.00

Is the statement "If S is a linearly dependent (ie NOT linearly independent) set then each vector is a linear combination of the other vectors in S" true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

- ○ True and I found a phrase and page number from the text
- ○ False and I can provide a counterexample
- ○ other

Check

Question **7**

Not complete

Points out of 1.00

Is the statement "If $\vec{x}$ and $\vec{y}$ are linearly independent and if $\vec{x}, \vec{y}, \vec{z}$ is linearly dependent (ie not linearly independent), then $\vec{z}$ is in the Span$\vec{x}, \vec{y}$" true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

- ○ True and I found a phrase and page number from the text
- ○ False and I can provide a counterexample
- ○ other

Check

Question **8**

Not complete

Points out of 1.00

Is the statement "If 3 vectors in $\mathbb{R}^3$ lie in the same plane, then they are linearly dependent (ie not linearly independent)" true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

- ○ True and I found a phrase and page number from the text
- ○ False and I can provide a counterexample
- ○ other

Check

Question **9**

Not complete

Points out of 1.00

Is the statement "If $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ are in $\mathbb{R}^3$ and $\vec{v}_3$ is not a linear combination of $\{\vec{v}_1, \vec{v}_2\}$ then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent" true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

- ○ True and I found a phrase and page number from the text
- ○ False and I can provide a counterexample
- ○ other

Check

Question **10**

Not complete

Points out of 1.00

Consider the following argument:

If $A\vec{x} = \vec{b}$ has two different solutions $\vec{v}_1$ and $\vec{v}_2$ so that $A\vec{v}_1 = \vec{b}$ and $A\vec{v}_2 = \vec{b}$ then

$A\vec{v}_1 - A\vec{v}_2 = \vec{b} - \vec{b}$ by substitution

$A(\vec{v}_1 - \vec{v}_2) = \vec{0}$ by factoring A and by the additive identity of $\vec{b}$

We also know that $\vec{v}_1 - \vec{v}_2 \neq \vec{0}$ since they were different solutions.

What does this show?

- the columns of A are linearly independent
- the columns of A are not linearly independent
- neither

Check

Question **11**

Not complete

Points out of 1.00

Which of the following are practical applications of linearly independent vectors as related to linear combinations?

- we can make more mixes with linearly independent vectors than if they weren't, because each vector provides another independent direction
- with independent vectors, any linear combination that can be made will be made uniquely with one set of weights rather than infinite possible weights, reducing the number of choices we might have to make
- both
- neither

What is the significance of the concept of span in coffee mixing?

- it shows us what mixes we can make
- it shows us what mixes we can not make
- both
- neither

Check

Question **12**

Not complete

Points out of 28.00

Given these vectors

$$\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

Set up the augmented matrix to solve for whether the vectors are linearly independent, preserving the ordering I used above:

☐	☐	☐	☐
☐	☐	☐	☐
☐	☐	☐	☐

Next reduce to Gaussian form using strict Gaussian and no row swaps. What is the row echelon form?

☐	☐	☐	☐
☐	☐	☐	☐
☐	☐	☐	☐

How many solutions does the system have?

- ○0
- ○1
- ○∞

Are the vectors linearly independent?

- ○yes
- ○no

What is the largest number of independent vectors we can find as a subset within this set of vector?

- ○3, the entire set
- ○2
- ○1

What is the span of the original vectors?

- ○an infinite volume

- ○ a plane
- ○ a line
- ○ a point

Check

Question **13**

Not complete

Points out of 1.00

To solidify and prepare for upcoming work, review and contemplate your knowledge and any questions that remain as related to definitions, concepts, computations, and examples from 1.7, including

- linearly independent set of vectors and connection to a homogeneous equation having only the trivial solution
- linearly dependent set of vectors and connection to nontrivial solutions existing and providing a dependance relation
- geometry of linearly independent set of 2 vectors: independent directions in space versus along the same line (Figure 1)
- geometry of linearly independent set of 3 or more vectors: no one vector is in the span of the rest, i.e. they are all needed to span the space versus redundancy in the geometric space they span in the sense that they aren't all needed to generate the same space under linear combinations (Figure 2)
- linearly independent columns of a matrix
- redundancy of $\vec{0}$ in a set of vectors $\{\vec{v}_1 = \vec{0}, \vec{v}_2, \ldots \vec{v}_n\}$ (Theorem 9)

Consider also 1.5, including

- algebra of homogeneous systems: $A\vec{x} = \vec{0}$
- algebraic solutions of homogenous systems always include the trivial solution=$\vec{0}$. nontrivial solutions, if any exist, are parameterized in parametric vector form using free variables to express those as well as the variables with pivots and then decomposed algebraically to showcase the algebra and geometry giving $t\vec{v}$ or $s\vec{u} + t\vec{v}$ or similar, where each free variable is attached to a vector.
- geometry of solutions of homogeneous systems are geometric spaces through the origin like lines, planes, or hyperplanes
- algebra of nonhomogeneous systens: $A\vec{x} = \vec{b}$
- solutions of nonhomogeneous systems in parametric vector form can be decomposed algebraically to showcase the algebra and geometry like $\vec{p} + t\vec{v}$, vectors ending on the line parallel to $\vec{v}$ or $\vec{p} + s\vec{u} + t\vec{v}$, vectors ending on the plane parallel to the one spanned by $\vec{u}, \vec{v}$...
- geometry of solutions of nonhomogeneous systems are geometric spaces translated away from the origin via adding $\vec{p}$

and 1.4, including

- algebra of matrix vector equation $A\vec{x} = \vec{b}$:
  - multiply a matrix and a column vector by linear combinations of the columns of *A* using weights from $\vec{x}$
  - span of the columns of *A* = set of all linear combinations of the columns of *A*
  - matrix vector equation $\longrightarrow$ vector equation $\longrightarrow$ augmented matrix
  - equations generic vectors $\vec{b}$ must satisfy to be in the span (Example 3)
  - dot products of rows of *A* with $\vec{x}$,
- geometry of solutions of matrix vector equation $A\vec{x} = \vec{b}$: spaces of subsets of R^3 spanned by the column vectors of A, geometry of such spaces (Figure 1)
- Theorem 4: relationship of consistency of $A\vec{x} = \vec{b}$ to always being a linear combination to spanning the entire R^m, where *m* is the number of rows, to having a pivot position in every row of *A*.
- identity matrix *I*

and 1.3, including

- algebra of vectors: coordinates, addition of vectors, scalar multiplication of vectors, properties like associativity under addition (property ii on p. 29), a linear combination with weights, zero vector, span of a set of vectors=all the linear combinations, is a vector in the span?, vector equation $\longrightarrow$ augmented matrix
- geometry of vectors in 2D and 3D: directed segment, parallelogram for addition, on same line for scalar multiplication of vectors, origin=zero vector, a linear combination geometrically in the plane or 3D, span=all the linear combinations geometrically in the plane or 3D, spaces of subsets of R^n spanned by vectors

and 1.2, including

- matrix of a linear system: row echelon form (Gaussian), reduced row echelon form (Gauss-Jordan)
- pivots: pivot position of a matrix, pivot column of a matrix
- row reduction algorithm we will most commonly use: elimination by forward phase and back substitution to row echelon form
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions with free variables and parametric solutions

and 1.1, including:

- algebra of linear equations: coefficients and variables
- geometry of linear equations in 2D and 3D: lines and planes
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions
- matrix of a linear system: coefficient matrix, augmented matrix, triangular form
- row equivalent systems
- algorithm for solving a linear system using elementary row operations of replacement, interchange, and scaling

When you have finished reviewing and reflecting, select one of the following (both receive full credit)

○ I currently have no questions

○ I will continue solidifying and understand that help is available in Dr. Sarah's more extensive feedback that follows below each question after I finish and open back up an entire practice quiz (this is more extensive than the hints that I can access during the open quiz), in Dr. Sarah's glossary/Wiki which is embedded into ASULearn from the linked terms, in Dr. Sarah's office hours and forum, and in Math Lab and Tutoring

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