Question **1**

Not complete

Points out of 5.00

Here are two images for you to consider.

Image 1

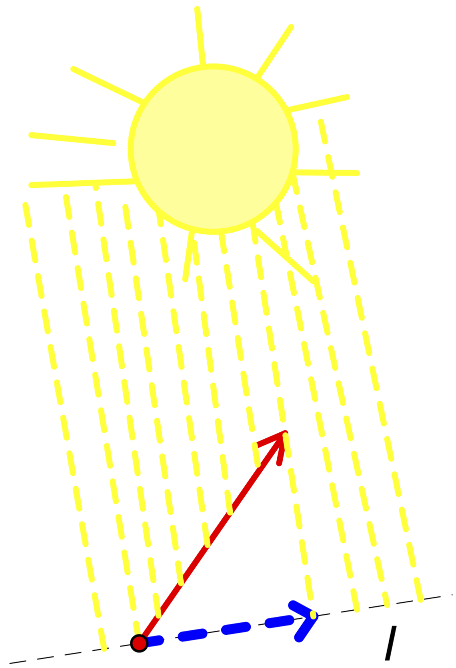
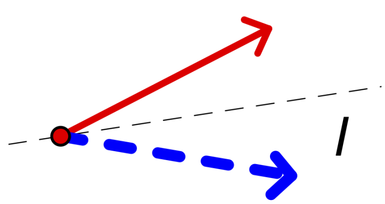


Image 2



If input is thin red and output is dashed blue, then which transformation does image 1 above showcase?

- reflection across the line *l*
- projection onto the line *l*
- other

If input is thin red and output is dashed blue, then which transformation does image 2 above showcase?

- reflection across the line *l*
- projection onto the line *l*
- other

The way that projection works in 2-space is that light rays or rain come in perpendicular to the line of projection. The shadow or dry spot is the projection of the input vector in solid red onto the output vector in dashed blue here. The way that reflection works in 2-space is that the line is the mirror and vectors are transformed so that the output are symmetric to the input with respect to the mirror.

Choose a different line than I did and sketch input and output diagrams for reflection across that line as well as projection onto that line. When you are finished with your sketches, type inputoutput in the box

What is similar about reflection across a line and projection onto a line l

○ both transformations fix vectors on the line l

○ there are no similarities

○ other

What is different about reflection across a line l versus projection onto a line l

○ all vectors that are projected end up on the line of projection while most vectors that are reflected end up off the line of reflection

○ there are no differences

○ other

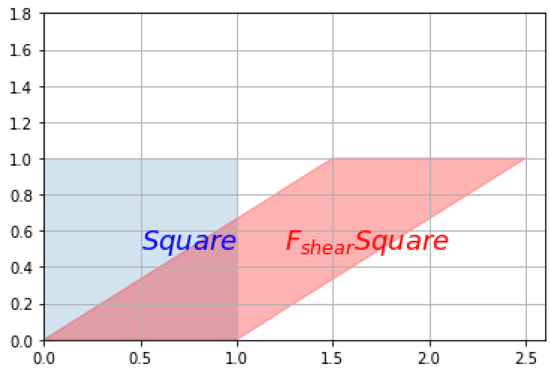
Check

Question **2**

Not complete

Points out of 1.00

Is the statement "The standard matrix of a horizontal shear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ has the form $\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$ where *a* and *d* are ± 1" true or false?



For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

○ True and I found a phrase and page number from the text

○ False and I can provide a counterexample

○ other

Check

Question **3**

Not complete

Points out of 1.00

How do our linear transformations act on $n \times 1$ column vectors $\vec{x}$ where $n > 1$ when the matrix representation A has more than one row.

- ○ left multiplication $x \to A\vec{x}$
- ○ right multiplication $x \to \vec{x}A$
- ○ both of the above

Check

Question **4**

Not complete

Points out of 5.00

Open
https://www.geogebra.org/m/w8888q8n
brothen_1 had created a reflection of the letter L and I modified it to make a very rough approximation of a PacMan figure. If I'd had a few more points to work with, it would have looked even better! The idea though is that we can approximate curved objects with linear pieces, which has wide application in computer graphics.

Drag the top PacMan, the red points, so that part of the figure overlaps with the line. That will help you visualize what stays fixed under this linear transformation of 2-space. What is fixed?

- there are no fixed points
- a point of reflection
- a line of reflection
- a plane of reflection

Next, put all the red points on the $y = -x$ line in quadrant 2 (i.e. (-1,1), (-2,2)...). What happens to the quadrant 2 red points on $y = -x$ under the reflection?

- they get flipped to quadrant 4 while staying on the y=-x line
- they go to quadrant 1
- they go to quadrant 3
- other

What matrix represents this reflection?

- $\begin{bmatrix} .5 & .5 \\ .5 & .5 \end{bmatrix}$
- $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- $\begin{bmatrix} cos(45°) & -sin(45°) \\ sin(45°) & cos(45°) \end{bmatrix}$

Open
https://www.geogebra.org/m/mqyqayru
Jonathan Holland, who has been at a number of universities as well as a research scientist at Compunetix, created a reflection in 3-space, which I modified. The figure being reflected has 3 perpendicular features---a cone, a cylinder, and a small sphere emanating from the central sphere so that you can better see the reflection features. Drag the points and turn the diagram so that you can actively engage with and solidify the visualization.

The lightly shaded red cylinder component has a point at the end of it. Drag it through the plane so that it intersects and you can see what is fixed by the reflection in 3-space. What is fixed?

- there are no fixed points
- a point of reflection
- a line of reflection
- a plane of reflection
- none of the above

Which could be the matrix representation of a reflection about a plane in 3-space?

- ○ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
- ○ $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- ○ both of the above

Check

Question **5**

Not complete

Points out of 8.00

Consider a dilation, the linear transformation $T(\vec{x}) = 3\vec{x}$ for vectors $\vec{x}$ in $\mathbb{R}^2$. We can find the matrix of this linear transformation by applying the linear transformation to the columns of the 2×2 Identity matrix to find the columns of the standard matrix for this linear transformation.

What is $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)$?

What is $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$?

Putting this together, we can find the standard matrix for the linear transformation since

$A\begin{bmatrix}1\\0\end{bmatrix} = 1 \cdot column1A + 0 \cdot column2A =$ column 1 of *A*

and similar for $A\begin{bmatrix}0\\1\end{bmatrix}$.

What is the standard matrix here?

Check

Question **6**

Not complete

Points out of 2.00

Open
https://www.geogebra.org/m/gctjpnys
which I modified from Juan Carlos Ponce Campuzano, University of Queensland, Australia

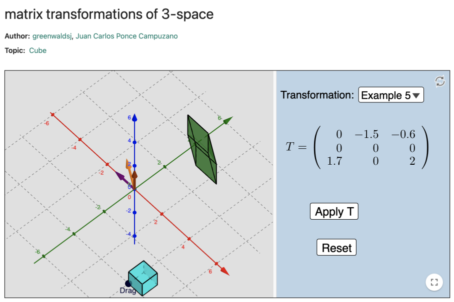
The vectors shown in the graph are the column vectors of the matrix, i.e. the image of the unit axes under the linear transformation.

Apply T to each of the five examples to see the impacts of the linear transformations on the cube. We use the drop down menu to change example and then Apply T to see the transformation. You can turn the graph to see it better if the transformed cube is out of sight.

What kind of linear transformations are Examples 1 and 2?

- ◯ dilation
- ◯ projection
- ◯ rotation
- ◯ reflection
- ◯ shear

Here is one view of Example 5 after the transformation:



After you select and Apply T in example 5 on https://www.geogebra.org/m/gctjpnys, turn the graph to line up all three column vectors in a "head on" view. Do this yourself to internalize the visualization. What does this show us?

- ◯ The column space, the span of the columns, is a plane, and the transformed cube is squished to a smaller space as it lies in this plane in 3-space.
- ◯ the transformation is a rotation
- ◯ both
- ◯ neither

Check

Question **7**

Not complete

Points out of 4.00

Open
https://www.geogebra.org/m/uct4xgv5
rm11821 has created matrix transformations of Leonardo da Vinci's Mona Lisa.

First modify the b slider to move it back and forth to see the effects of a horizontal shear on the figure as the corresponding matrix changes. What stays fixed under this transformation?

- vectors on the x-axis
- vectors on the y-axis
- vectors on another line
- only the origin

Then use the Reset button to return to the identity matrix. Next, use the a slider to make $a = -1$, a reflection. What stays fixed?

- vectors on the x-axis
- vectors on the y-axis
- vectors on another line
- only the origin

Next use the d slider to make $d = -1$, so that both $a = -1$ and $d = -1$. What stays fixed?

- vectors on the x-axis
- vectors on the y-axis
- vectors on another line
- only the origin

What transformation is this?

- projection
- rotation
- reflection about a line
- shear

Check

Question **8**

Not complete

Points out of 4.00

To visualize how a matrix acts as a linear transformation, we can consider the columns of the matrix as the outputs of the unit axes and sketch input/output diagrams to help visualize, test out other points, and/or match the matrix to listing of transformations, like in the reading.

The linear transformation $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ is a

- ○ reflection about a line
- ○ counterclockwise rotation by π about the origin
- ○ vertical shear
- ○ projection onto a line

What does the linear transformation $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ fix and what does it do to other vectors in the plane?

- ○ fixes vectors on the line of reflection and reflects any other vectors across that line
- ○ rotates any vector 180 degrees from where it was originally pointing
- ○ fixes vectors on the $y-$axis and blows anything else with the wind of the shear
- ○ fixes vectors on the line of projection and sends any other vector to its shadow on the line of projection via light rays that come in perpendicular to the line of projection

The linear transformation $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ is a

- ○ reflection about a line
- ○ counterclockwise rotation by π about the origin
- ○ vertical shear
- ○ projection onto a line

What does the linear transformation $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ fix and what does it do to other vectors in the plane?

- ○ fixes vectors on the line of reflection and reflects any other vectors across that line
- ○ rotates any vector 180 degrees from where it was originally pointing
- ○ fixes vectors on the $y-$axis and blows anything else with the wind of the shear
- ○ fixes vectors on the line of projection and sends any other vector to its shadow on the line of projection via light rays that come in perpendicular to the line of projection

Check

Question **9**

Not complete

Points out of 3.00

Match each matrix A with the geometric description of the linear transformation $T(\vec{x}) = A\vec{x}$.

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

- ○ projects a vector in $\mathbb{R}^3$ onto the *xy*-plane in $\mathbb{R}^3$.
- ○ horizontal shear like wind blowing horizontally with a fixed base on the *x*-axis.
- ○ rotates a vector counterclockwise about the origin through 90 degrees.
- ○ other

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

- ○ projects a vector in $\mathbb{R}^3$ onto the *xy*-plane in $\mathbb{R}^3$.
- ○ horizontal shear like wind blowing horizontally with a fixed base on the *x*-axis.
- ○ rotates a vector counterclockwise about the origin through 90 degrees.
- ○ other

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- ○ projects a vector in $\mathbb{R}^3$ onto the *z=0* plane or equivalently the *xy0*-plane in $\mathbb{R}^3$.
- ○ horizontal shear like wind blowing horizontally with a fixed base on the *x*-axis.
- ○ rotates a vector counterclockwise about the origin through 90 degrees.
- ○ other

Check

Question **10**

Not complete

Points out of 1.00

Is the statement "If *A* is a *4x3* matrix then the transformation $\vec{x} \to A\vec{x}$ maps $\mathbb{R}^3$ onto $\mathbb{R}^4$ so that the range of the linear transformation is all of $\mathbb{R}^4$" true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the text.

- ○ True and I found a phrase and page number from the text
- ○ False and I can provide a counterexample
- ○ other

Check

Question **11**

Not complete

Points out of 3.00

Which linear transformations are invertible?

To obtain credit, select all that apply

- ☐ dilation by a nonzero factor
- ☐ projection
- ☐ reflection
- ☐ rotation by θ
- ☐ shear

Write down a 2×2 matrix that represents a linear transformation of the plane that is not invertible. What is its determinant?

- ○ 1
- ○ 0
- ○ other

Check

Question **12**

Not complete

Points out of 5.00

Look at the linear transformation $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

What kind of linear transformation is this?

- ○ dilation
- ○ projection
- ○ reflection
- ○ rotation
- ○ shear

What is the column space?

- ○ $\vec{0}$
- ○ a line through the origin
- ○ $\mathbb{R}^2$

What is the rank?

[ ]

What is the null space?

- ○ $\vec{0}$
- ○ a line through the origin
- ○ $\mathbb{R}^2$

What is rank + nullity equal to?

[ ]

Check

Question **13**

Not complete

Points out of 1.00

To solidify and prepare for upcoming work, review and contemplate your knowledge and any questions that remain as related to definitions, concepts, computations, and examples from 1.8 and 1.9, including

- linear transformation: addition and scalar multiplication
- left multiplication matrix representations
- dilation, projection, reflection, rotation, shear (see Examples 2-5 in 1.8, Examples 2-3 in 1.9, and tables 1-4 in 2.8)
- the algebraic image of the unit axes as a way to find the matrix of the transformation
- range of a linear transformation: the algebraic or geometric images or outputs, e.g. of the unit square as a way to visualize the transformation and understand its effects
- the range, image or output of a sheared sheep

Since the material builds on itself, consider whether there is any material from before this module that you want to brush up on:

1.1

- algebra of linear equations: coefficients and variables
- geometry of linear equations in 2D and 3D: lines and planes
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions
- matrix of a linear system: coefficient matrix, augmented matrix, triangular form
- row equivalent systems
- algorithm for solving a linear system using elementary row operations of replacement, interchange, and scaling

1.2

- matrix of a linear system: row echelon form (Gaussian), reduced row echelon form (Gauss-Jordan)
- pivots: pivot position of a matrix, pivot column of a matrix
- row reduction algorithm we will most commonly use: elimination by forward phase and back substitution to row echelon form
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions with free variables and parametric solutions

1.3

- algebra of vectors: coordinates, addition of vectors, scalar multiplication of vectors, properties like associativity under addition (property ii on p. 29), a linear combination with weights, zero vector, span of a set of vectors=all the linear combinations, is a vector in the span?, vector equation $\longrightarrow$ augmented matrix
- geometry of vectors in 2D and 3D: directed segment, parallelogram for addition, on same line for scalar multiplication of vectors, origin=zero vector, a linear combination geometrically in the plane or 3D, span=all the linear combinations geometrically in the plane or 3D, spaces of subsets of R^n spanned by vectors

1.4

- algebra of matrix vector equation $A\vec{x} = \vec{b}$:
  - multiply a matrix and a column vector by linear combinations of the columns of *A* using weights from $\vec{x}$
  - span of the columns of *A* = set of all linear combinations of the columns of *A*
  - matrix vector equation $\longrightarrow$ vector equation $\longrightarrow$ augmented matrix
  - equations generic vectors $\vec{b}$ must satisfy to be in the span (Example 3)
  - dot products of rows of *A* with $\vec{x}$,
- geometry of solutions of matrix vector equation $A\vec{x} = \vec{b}$: spaces of subsets of R^3 spanned by the column vectors of A, geometry of such spaces (Figure 1)
- Theorem 4: relationship of consistency of $A\vec{x} = \vec{b}$ to always being a linear combination to spanning the entire R^m, where *m* is the number of rows, to having a pivot position in every row of *A*.
- identity matrix *I*

1.5

- algebra of homogeneous systems: $A\vec{x} = \vec{0}$
- algebraic solutions of homogenous systems always include the trivial solution=$\vec{0}$. nontrivial solutions, if any exist, are parameterized in parametric vector form using free variables to express those as well as the variables with pivots and then decomposed algebraically to showcase the algebra and geometry giving $t\vec{v}$ or $s\vec{u} + t\vec{v}$ or similar, where each free variable is attached to a vector.

- geometry of solutions of homogeneous systems are geometric spaces through the origin like lines, planes, or hyperplanes
- algebra of nonhomogeneous systens: $A\vec{x} = \vec{b}$
- solutions of nonhomogeneous systems in parametric vector form can be decomposed algebraically to showcase the algebra and geometry like $\vec{p} + t\vec{v}$, vectors ending on the line parallel to $\vec{v}$ or $\vec{p} + s\vec{u} + t\vec{v}$, vectors ending on the plane parallel to the one spanned by $\vec{u}, \vec{v}$...
- geometry of solutions of nonhomogeneous systems are geometric spaces translated away from the origin via adding $\vec{p}$

1.7

- linearly independent set of vectors and connection to a homogeneous equation having only the trivial solution
- linearly dependent set of vectors and connection to nontrivial solutions existing and providing a dependance relation
- geometry of linearly independent set of 2 vectors: independent directions in space versus along the same line (Figure 1)
- geometry of linearly independent set of 3 or more vectors: no one vector is in the span of the rest, i.e. they are all needed to span the space versus redundancy in the geometric space they span in the sense that they aren't all needed to generate the same space under linear combinations (Figure 2)
- linearly independent columns of a matrix
- redundancy of $\vec{0}$ in a set of vectors $\{\vec{v}_1 = \vec{0}, \vec{v}_2, \ldots \vec{v}_n\}$ (Theorem 9)

2.1

- matrices: diagonal matrix [and main diagonal], zero matrix
- matrix operations: matrix addition, scalar multiplication of a matrix, matrix multiplication, powers of a matrix, left (or right) multiplication, transpose of a matrix
- matrix multiplication by linear combinations of the columns of A using weights from the corresponding column of B or by the dot products of a row of A with the corresponding column of B.
- algebraic properties that do hold for matrix multiplication: associativity and one-sided distributivity
- algebraic properties that don't hold for matrix multiplication: commutativity

2.2

- matrices: invertible (nonsingular) matrix, noninvertible (singular) matrix, elementary matrix
- determinant and inverse of a 2x2 matrix
- connection between invertibility and unique solutions
- inverse of a product of matrices and inverse of a transpose

2.3

- what makes a matrix invertible for a square matrix (Theorem 8 statements aside from f. and i., which we haven't covered)
- condition number (numerical note on p. 123)

2.8

- subspace properties: closed under addition and scalar multiplication
- spaces associated to a matrix: column space and null space
- basis: linearly independent spanning set
- basis for column space as the pivot columns
- basis for null space as the vectors attached to free variables in parametric solutions of the homogeneous system $A\vec{x} = \vec{0}$

2.9

- dimension of a space
- rank of a matrix (dimension of column space)
- nullity of a matrix (dimension of null space)
- rank nullity theorem (Theorem 14)
- what makes a matrix invertible continued: adding rank and nullity to Theorem 8 when the matrix is square

When you have finished reviewing and reflecting, select one of the following (both receive full credit)

○ I currently have no questions

○ I will continue solidifying and understand that help is available in Dr. Sarah's more extensive feedback that follows below each question after I finish and open back up an entire practice quiz (this is more extensive than the hints that I can access during the open quiz), in Dr. Sarah's glossary/Wiki which is embedded into ASULearn from the linked terms, in Dr. Sarah's office hours and forum, and in Math Lab and Tutoring

Check