Question 1

Not complete

Points out of 2.00

To shore up the two different matrix multiplication methods, recall that by-hand we can either multiply the first matrix by each successive column vector (multiply a matrix and a column vector) which can then be performed by the linear combinations weight method, or we can use the dot products of the rows of the first matrix with the columns of the second matrix method.

Multiply the first matrix by each successive column vector $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

What do we obtain?

- ○ $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 2 \\ 4 \end{bmatrix}$
- ○ $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$
- ○ other

Next multiply the matrix times each vector (multiply a matrix and a column vector) to fine the complete product. What do we obtain?

- ○ $\begin{bmatrix} a & 2b \\ 3c & 4d \end{bmatrix}$
- ○ $\begin{bmatrix} a+2b & 3a+4b \\ c+2d & 3c+4d \end{bmatrix}$
- ○ $\begin{bmatrix} a+3b & 2a+4b \\ c+3d & 2c+4d \end{bmatrix}$
- ○ other

For this product, $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix}$

use the dot product method of the row of the matrix on the left with the column of the matrix on the right for matrix multiplication. What is the resulting entry in row 1 column 2?

- ○ the multiplication is not defined as the matrices are the wrong size
- ○ 1×8 + 2×8 + 3×8
- ○ 4×7 + 5×9 + 6×11
- ○ 1×8 + 2×10 + 3×12
- ○ other

What size matrix is $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix}$

- ○ the multiplication is not defined as the matrices are the wrong size
- ○ 2x3
- ○ 3x2
- ○ 3x3
- ○ 2x2

What size matrix is $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix}$

- ○ the addition is not defined as the matrices are the wrong size
- ○ 2x3
- ○ 3x2
- ○ 3x3
- ○ 2x2

Check

Question **2**

Not complete

Points out of 6.00

Let $A = \begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 \\ 3 & 4 \end{bmatrix}$, and $C = \begin{bmatrix} -3 & -5 \\ 2 & 1 \end{bmatrix}$

Part a) Compute *AB* (Choose a method to compute matrix multiplication (either multiply the first matrix by each successive vector (multiply a matrix and a column vector) which can then be performed by the linear combinations weight method, or use the dot products of the rows of the first matrix with the cols of the second method). Which of the following is *AB*?

- ○ $\begin{bmatrix} 3(-1) & -6(1) \\ -1(3) & 2(4) \end{bmatrix}$
- ○ $\begin{bmatrix} 3(-1)-6(1) & 3(3)-6(4) \\ -1(-1)+2(1) & -1(3)+2(4) \end{bmatrix}$
- ○ $\begin{bmatrix} 3(-1)-6(3) & 3(1)-6(4) \\ -1(-1)+2(3) & -1(1)+2(4) \end{bmatrix}$
- ○ other

Part b) Next compute *AC* by hand, using whichever definition of matrix multiplication you like. *S*implify your calculation so that you have a simplified number in each spot (like -5), with no extra spaces or characters.

[ ]	[ ]
[ ]	[ ]

Part c) Is it true that in this case that *AB=AC*, yet *B* is not equal to *C*?

- ○ True
- ○ False

Check

Question **3**

Not complete

Points out of 6.00

What values of *k*, if any, will make *AB*=*BA* where $A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 9 \\ -3 & k \end{bmatrix}$.

Part a) Compute *AB* (Choose a method to compute matrix multiplication (either multiply the first matrix by each successive vector (multiply a matrix and a column vector) which can then be performed by the linear combinations weight method, or use the dot products of the rows of the first matrix with the cols of the second method). Which of the following is *AB*?

- ○ $\begin{bmatrix} 2(1) & 3(9) \\ -1(-3) & 1(k) \end{bmatrix}$
- ○ $\begin{bmatrix} 2(1)+3(-3) & 2(9)+3(k) \\ -1(1)+1(-3) & -1(9)+1(k) \end{bmatrix}$
- ○ $\begin{bmatrix} 2(1)+3(9) & 2(-3)+3(k) \\ -1(1)+1(9) & -1(-3)+1(k) \end{bmatrix}$
- ○ other

Part b) Next compute *BA* by hand, using whichever definition of matrix multiplication you like. Simplify your calculation so that you have a simplified number or expression in each spot (like -25-k), with no extra spaces or characters.

☐	☐
☐	☐

Part c) What values of *k*, if any, will make *AB*=*BA*?

- ○ *k*=1
- ○ *k*=-2
- ○ no values work
- ○ any *k* works
- ○ other

Check

Question **4**

Not complete

Points out of 1.00

Is the statement "Each column of *AB* is a linear combination of the columns of *B* using weights from the corresponding columns of *A*" true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

- ○ True and I found a phrase and page number from the text
- ○ False and I can provide a counterexample
- ○ other

Check

Question **5**

Not complete

Points out of 1.00

Is the statement "The transpose of a product of matrices equals the product of their transposes in the same order " true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

- ◯ True and I found a phrase and page number from the text
- ◯ False and I can provide a counterexample
- ◯ other

Check

Question **6**

Not complete

Points out of 2.00

Suppose the last column of AB is entirely zero but B itself has no column of zeros.

Part a) What can you say about the columns of A?

- ◯ The columns of A must be the identity matrix columns
- ◯ A must have entries of zero
- ◯ The columns of A are not linearly independent.
- ◯ The columns of A must span the entire space

Part b) Which definition of matrix multiplication is needed to reason here?

- ◯ A times the columns of B method
- ◯ Dot products of the rows of A with the cols of B method

Part c) Write out your reasoning of how Part b) can be used to show Part a) (and compare it with the general feedback, which is available after you finish and submit the entire practice quiz).

Check

Question **7**

Not complete

Points out of 4.00

Let $\vec{u} = \begin{bmatrix} -3 \\ 2 \\ -5 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

Part a) What is $\vec{u}^T\vec{v}$, where $\vec{u}^T$ is the transpose?

- ○ -3a+2b-5c
- ○ $\begin{bmatrix} -3a & -3b & -3c \\ 2a & 2b & 2c \\ -5a & -5b & -5c \end{bmatrix}$
- ○ $\begin{bmatrix} -3a & 2a & -5a \\ -3b & 2b & -5b \\ -3c & 2c & -5c \end{bmatrix}$
- ○ other

Part b) What is $\vec{v}^T\vec{u}$?

- ○ -3a+2b-5c
- ○ $\begin{bmatrix} -3a & -3b & -3c \\ 2a & 2b & 2c \\ -5a & -5b & -5c \end{bmatrix}$
- ○ $\begin{bmatrix} -3a & 2a & -5a \\ -3b & 2b & -5b \\ -3c & 2c & -5c \end{bmatrix}$
- ○ other

Part c) What is $\vec{u}\vec{v}^T$?

- ○ -3a+2b-5c
- ○ $\begin{bmatrix} -3a & -3b & -3c \\ 2a & 2b & 2c \\ -5a & -5b & -5c \end{bmatrix}$
- ○ $\begin{bmatrix} -3a & 2a & -5a \\ -3b & 2b & -5b \\ -3c & 2c & -5c \end{bmatrix}$
- ○ other

Part d) What is $\vec{v}\vec{u}^T$?

- ○ -3a+2b-5c
- ○ $\begin{bmatrix} -3a & -3b & -3c \\ 2a & 2b & 2c \\ -5a & -5b & -5c \end{bmatrix}$
- ○ $\begin{bmatrix} -3a & 2a & -5a \\ -3b & 2b & -5b \\ -3c & 2c & -5c \end{bmatrix}$
- ○ other

Multiplying by the transpose will be helpful in least squares regression!

Check

Question **8**

Not complete

Points out of 24.00

Choose a method to compute by-hand matrix multiplication (either multiply the first matrix by each successive column vector (multiply a matrix and a column vector) which can then be performed by the linear combinations weight method, or use the dot products of the rows of the first matrix with the cols of the second matrix method).

Write the product of $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} -2 & -12 \\ -5 & -13 \end{bmatrix}$ in your notes.

What is row 1 column 1 of the product $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} -2 & -12 \\ -5 & -13 \end{bmatrix}$?

- ○-2a
- ○-2a-5b
- ○-2a-12c
- ○other

What is row 1 column 2 of the product $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} -2 & -12 \\ -5 & -13 \end{bmatrix}$?

- ○-12b
- ○-5a-13c
- ○-12a-13b
- ○other

What is row 2 column 1 of the product $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} -2 & -12 \\ -5 & -13 \end{bmatrix}$?

- ○-12b
- ○-12a-13b
- ○-5a-13c
- ○other

What is row 2 column 2 of the product $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} -2 & -12 \\ -5 & -13 \end{bmatrix}$?

- ○-13d
- ○-12c-13d
- ○-5c-13d
- ○other

Set the product matrix entries equal to $\begin{bmatrix} 3 & 1 \\ 8 & 14 \end{bmatrix}$ and obtain a system of 4 equations in the 4 unknowns a, b, c, d. Some of the coefficients will be zero. What is the augmented matrix for this system when the items are defined as below:

	coefficient of a	coefficient of b	coefficient of c	coefficient of d	=
row 1 column 1 of the product matrix					
row 1 column 2 of the product matrix					
row 2 column 1 of the product matrix					
row 2 column 2 of the product matrix					

This kind of algebra will be helpful when we work on decoding the Hill Cipher in the linear transformations section!

Check

Question **9**

Not complete

Points out of 1.00

To solidify and prepare for upcoming work, review and contemplate your knowledge and any questions that remain as related to definitions, concepts, computations, and examples from 2.1, including

- matrices: diagonal matrix [and main diagonal], zero matrix
- matrix operations: matrix addition, scalar multiplication of a matrix, matrix multiplication, powers of a matrix, left (or right) multiplication, transpose of a matrix
- matrix multiplication by linear combinations of the columns of A using weights from the corresponding column of B or by the dot products of a row of A with the corresponding column of B.
- algebraic properties that do hold for matrix multiplication: associativity and one-sided distributivity
- algebraic properties that don't hold for matrix multiplication: commutativity

Since the material builds on itself, consider whether there is any material from before this module that you want to brush up on:

1.1

- algebra of linear equations: coefficients and variables
- geometry of linear equations in 2D and 3D: lines and planes
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions
- matrix of a linear system: coefficient matrix, augmented matrix, triangular form
- row equivalent systems
- algorithm for solving a linear system using elementary row operations of replacement, interchange, and scaling

1.2

- matrix of a linear system: row echelon form (Gaussian), reduced row echelon form (Gauss-Jordan)
- pivots: pivot position of a matrix, pivot column of a matrix
- row reduction algorithm we will most commonly use: elimination by forward phase and back substitution to row echelon form
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions with free variables and parametric solutions

1.3

- algebra of vectors: coordinates, addition of vectors, scalar multiplication of vectors, properties like associativity under addition (property ii on p. 29), a linear combination with weights, zero vector, span of a set of vectors=all the linear combinations, is a vector in the span?, vector equation $\longrightarrow$ augmented matrix
- geometry of vectors in 2D and 3D: directed segment, parallelogram for addition, on same line for scalar multiplication of vectors, origin=zero vector, a linear combination geometrically in the plane or 3D, span=all the linear combinations geometrically in the plane or 3D, spaces of subsets of R^n spanned by vectors

1.4

- algebra of matrix vector equation $A\vec{x} = \vec{b}$:
  - multiply a matrix and a column vector by linear combinations of the columns of *A* using weights from $\vec{x}$
  - span of the columns of *A* = set of all linear combinations of the columns of *A*
  - matrix vector equation $\longrightarrow$ vector equation $\longrightarrow$ augmented matrix
  - equations generic vectors $\vec{b}$ must satisfy to be in the span (Example 3)
  - dot products of rows of *A* with $\vec{x}$,
- geometry of solutions of matrix vector equation $A\vec{x} = \vec{b}$: spaces of subsets of R^3 spanned by the column vectors of A, geometry of such spaces (Figure 1)
- Theorem 4: relationship of consistency of $A\vec{x} = \vec{b}$ to always being a linear combination to spanning the entire R^m, where *m* is the number of rows, to having a pivot position in every row of *A*.
- identity matrix *I*

1.5

- algebra of homogeneous systems: $A\vec{x} = \vec{0}$
- algebraic solutions of homogenous systems always include the trivial solution=$\vec{0}$. nontrivial solutions, if any exist, are parameterized in parametric vector form using free variables to express those as well as the variables with pivots and then decomposed algebraically to showcase the algebra and geometry giving $t\vec{v}$ or $s\vec{u} + t\vec{v}$ or similar, where each free variable is attached to a vector.

- geometry of solutions of homogeneous systems are geometric spaces through the origin like lines, planes, or hyperplanes
- algebra of nonhomogeneous systens: $A\vec{x} = \vec{b}$
- solutions of nonhomogeneous systems in parametric vector form can be decomposed algebraically to showcase the algebra and geometry like $\vec{p} + t\vec{v}$, vectors ending on the line parallel to $\vec{v}$ or $\vec{p} + s\vec{u} + t\vec{v}$, vectors ending on the plane parallel to the one spanned by $\vec{u}, \vec{v}$...
- geometry of solutions of nonhomogeneous systems are geometric spaces translated away from the origin via adding $\vec{p}$

1.7

- linearly independent set of vectors and connection to a homogeneous equation having only the trivial solution
- linearly dependent set of vectors and connection to nontrivial solutions existing and providing a dependance relation
- geometry of linearly independent set of 2 vectors: independent directions in space versus along the same line (Figure 1)
- geometry of linearly independent set of 3 or more vectors: no one vector is in the span of the rest, i.e. they are all needed to span the space versus redundancy in the geometric space they span in the sense that they aren't all needed to generate the same space under linear combinations (Figure 2)
- linearly independent columns of a matrix
- redundancy of $\vec{0}$ in a set of vectors $\{\vec{v}_1 = \vec{0}, \vec{v}_2, \ldots \vec{v}_n\}$ (Theorem 9)

When you have finished reviewing and reflecting, select one of the following (both receive full credit)

○ I currently have no questions

○ I will continue solidifying and understand that help is available in Dr. Sarah's more extensive feedback that follows below each question after I finish and open back up an entire practice quiz (this is more extensive than the hints that I can access during the open quiz), in Dr. Sarah's glossary/Wiki which is embedded into ASULearn from the linked terms, in Dr. Sarah's office hours and forum, and in Math Lab and Tutoring

Check