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As a review, via the linear combination def of mult of A with a column vector: $b_{1n}\text{col}1A + \ldots + b_{nn}\text{col}nA = \vec{0}$. We have at least one non-zero $b_{\_\_n}$ since B has no col of 0s (i.e. a non-trivial weight for $A\vec{x} = \vec{0}$), so the columns of A are not l.i

Applying the inverse (if it exists)—2 Common Methods

Multiply both sides: $A^{-1}(A\vec{x}) = A^{-1}\vec{b}$
Reorder by associativity: $(A^{-1}A)\vec{x} = A^{-1}\vec{b}$
Cancel A by its inverse: $I_{n\times n}\vec{x} = A^{-1}\vec{b}$
Reduce identity: $\vec{x} = A^{-1}\vec{b}$

OR

$A_{n\times n}$ must have full pivots to be invertible because it reduces to the identity matrix I so you can make use of the full pivots