Determinants

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \xrightarrow{\text{Method 1: First 2 columns/6 diagonals}} \begin{matrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{matrix}$$

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3 main diagonals: $a \cdot e \cdot i + b \cdot f \cdot g + c \cdot d \cdot h$
minus 3 off diagonals: $-c \cdot e \cdot g - a \cdot f \cdot h - b \cdot d \cdot i$

2×2 has 2 terms, 3×3 has 6 terms, 4×4 has 24 terms. Do you see a pattern?

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$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \xrightarrow{\text{Method 2: Laplace expansion (along any row or column)}}$$

$$\sum_{1}^{n} a_{ij} \cdot (-1)^{i+j} \cdot \text{Det of matrix obtained by eliminating row i and column j}$$

where we have fixed i or j as the row or column we are expanding along. For example:
$\sum_{1}^{n} a_{2j} \cdot (-1)^{2+j} \cdot$ Det of matrix obtained by eliminating row 2 and column j

$$= a_{21} \cdot (-1)^{2+1} \begin{vmatrix} b & c \\ h & i \end{vmatrix} + a_{22} \cdot (-1)^{2+2} \begin{vmatrix} a & c \\ g & i \end{vmatrix} + a_{23} \cdot (-1)^{2+3} \begin{vmatrix} a & b \\ g & h \end{vmatrix}$$

$$= d \cdot (-1)^{2+1} \begin{vmatrix} b & c \\ h & i \end{vmatrix} + e \cdot (-1)^{2+2} \begin{vmatrix} a & c \\ g & i \end{vmatrix} + f \cdot (-1)^{2+3} \begin{vmatrix} a & b \\ g & h \end{vmatrix}$$