EBG # 3 Using Gaussian Elimination (Echelon Form)

Gaussian Elimination: 0s below the main diagonal

Augmented matrix: $\begin{matrix} x & y & = \end{matrix}$ $\begin{bmatrix} 1 & 1 & 17 \\ 4 & 2 & 48 \end{bmatrix}$

(*Replacement*) Replace a row by the sum of itself and a multiple of another row:

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- ~~[0 0 nonzero] (0x+0y=nonzero) *inconsistent*, 0 concurrent sols~~.
- If *consistent*, use *pivots* = 1st nonzero entry in a row. 1 is a pivot for x, -2 is a pivot for y. Any variables without pivots are free. Full pivots here—row 2: y=10 and we can use back substitution (row 1: x+y=17, so x=17-y=17-10=7), so (7, 10) is *unique* solution (point in $\mathbb{R}^2$ -intersection of lines)

Elementary Row Operations

- (Interchange) Swap two equations
- (Scaling) Multiply an equation by a non-zero constant
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- Save the x term in eq 1 and use it to eliminate all the other x terms below it via $r_k' = cr_1 + r_k$
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Critical Analysis–Solutions

- [0 0 0 ... 0 nonzero] inconsistent.
- If the system is consistent then the last row with non-zero coefficients will yield $x_k = b$, and then solve using back substitution (any variables without pivots are free).

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$x + ky = 0$
$kx + y = 0$
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Use x term in eq1 to eliminate terms below it via $r'_k = cr_1 + r_k$

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Case 2: $-k^2 + 1 =$ nonzero: full pivots. row 2: nonzero$y = 0$ so $y = 0$. row 1 $x + ky = 0$ so $x = 0$. Solution: (0,0)