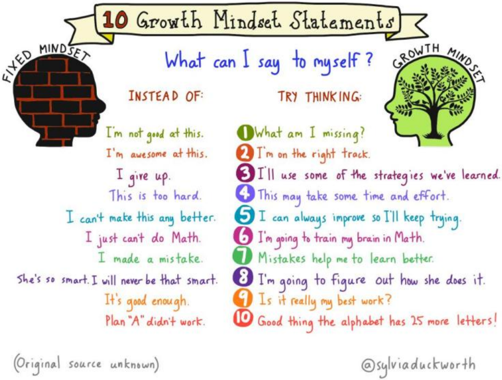
1. Multiplying a column vector $\vec{v}_1$ by a real number c_1

a) scales each entry algebraically, but has no geometric interpretation

b) keeps $\vec{v}_1$ on the same line through the origin and stretches or shrinks it according to the value of c_1

c) creates a diagonal of the parallelogram formed by $\vec{v}_1$ and c_1

d) none of the above



2. What do the collection of column vectors $c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, for c_1 and c_2 real, have in common?

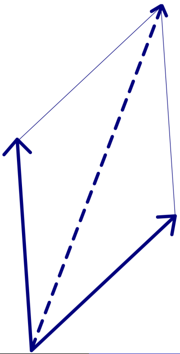
a) They are vectors of the form $\begin{bmatrix} c_1 + 2c_2 \\ c_1 + 2c_2 \end{bmatrix}$

b) They create the diagonals of parallelograms

c) They form all of $\mathbb{R}^2$

d) both a) and b)

e) both a) and c)



3. Notice that $-1 \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} + 2 \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$. More generally, what do the collection of column vectors

$$c_1 \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$$, for c_1 and c_2 real, have in common geometrically?

a) the line connecting the tips of $\begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$

b) the plane formed by $\begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$

c) a non-linear curve

d) a non-linear surface

e) none of the above

4. A coffee shop offers two blends of coffees: House & Deluxe. Each is a blend of Brazil, Colombia, Kenya, and Sumatra roasts: $\begin{bmatrix} & \text{House} & \text{Deluxe} \\ \text{Brazil} & 30\% & 40\% \\ \text{Columbia} & 20\% & 30\% \\ \text{Kenya} & 20\% & 20\% \\ \text{Sumatra} & 30\% & 10\% \end{bmatrix}$.

Suppose that the shop has 36 lbs of Brazil roast, 26 lbs of Columbia roast, 20 lbs of Kenya roast, and 18 lbs of Sumatra roast in stock and wants to completely use up the stock of coffee at hand in making the blends. What represents the system?

a) $\text{lbs House} \begin{bmatrix} .3 \\ .2 \\ .2 \\ .3 \end{bmatrix} + \text{lbs Deluxe} \begin{bmatrix} .4 \\ .3 \\ .2 \\ .1 \end{bmatrix} = \begin{bmatrix} 36 \\ 26 \\ 20 \\ 18 \end{bmatrix}$

b) $\begin{bmatrix} .3 & .4 \\ .2 & .3 \\ .2 & .2 \\ .3 & .1 \end{bmatrix} \begin{bmatrix} \text{lbs House} \\ \text{lbs Deluxe} \end{bmatrix} = \begin{bmatrix} 36 \\ 26 \\ 20 \\ 18 \end{bmatrix}$

c) $\begin{bmatrix} .3\text{lbs House} + .4\text{lbs Deluxe} \\ .2\text{lbs House} + .3\text{lbs Deluxe} \\ .2\text{lbs House} + .2\text{lbs Deluxe} \\ .3\text{lbs House} + .1\text{lbs Deluxe} \end{bmatrix} \begin{bmatrix} \text{lbs House} \\ \text{lbs Deluxe} \end{bmatrix} = \begin{bmatrix} 36 \\ 26 \\ 20 \\ 18 \end{bmatrix}$

d) two responses from above

e) a) b) and c)

5. We perform the following in Maple:

```
s13n15extension:=Matrix([[1,-5,b1],[3,-8,b2],[-1,2,b3]]);
ReducedRowEchelonForm(s13n15extension);
```

and obtain the 3x3 identity $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Which are true?

a) $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is never in the span of $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} -5 \\ -8 \\ 2 \end{bmatrix}$

b) $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is never a linear combination of $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} -5 \\ -8 \\ 2 \end{bmatrix}$

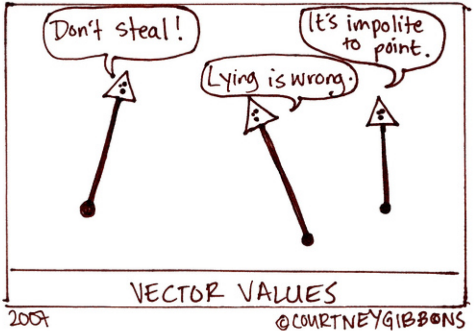
c) $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is never in the plane formed by $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} -5 \\ -8 \\ 2 \end{bmatrix}$

d) all of the above

e) none of the above

6. For two column vectors $\vec{v}_1$ and $\vec{v}_2$,
$\{c_1\vec{v}_1 + \vec{v}_2$ so that c_1 is real$\}$ is

a) a collection of vectors whose tips lie on the line parallel to $\vec{v}_1$ and through the tip of $\vec{v}_2$
b) a collection of vectors whose tips lie on the line parallel to $\vec{v}_2$ and through the tip of $\vec{v}_1$
c) a line because c_1 is free, but we can't say any more about it
d) more than one of the above



http://brownsharpie.courtneygibbons.org/wp-content/comics/2007-12-28-vector-values-1.jpg

7. Suppose that the augmented matrix for a system reduces to $\begin{bmatrix} 1 & -4 & 5 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Describe the solutions, the intersections of the rows, geometrically and in parametric vector form.

a) a line $t \begin{bmatrix} 1 \\ -4 \\ 5 \\ 6 \end{bmatrix}$ in $\mathbb{R}^4$

b) a plane $s \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$ in $\mathbb{R}^3$

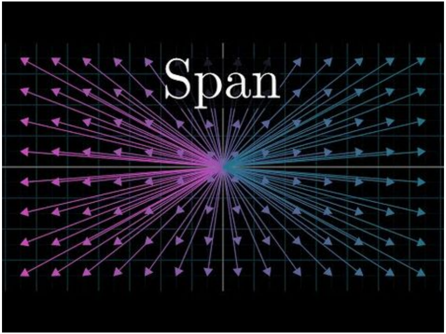
c) another line

d) another plane

e) none of the above

8. Compare the span of the 3 vectors $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, to the span of the 2 vectors $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

a) The spans are the same and I have a good reason why
b) The spans are the same but I am unsure of why
c) The spans are different but I am unsure of why
d) The spans are different and I have a good reason why



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How to express the redundancy? 1.7:
The vectors $\vec{v}_1, \vec{v}_2, ..., \vec{v}_n$ are linearly independent (l.i.), if and only if: $c_1\vec{v}_1 + c_2\vec{v}_2 + ... + c_n\vec{v}_n = \vec{0}$ has only the trivial solution (ie $c_i = 0$).

9. What can we say about the pivots for the augmented matrix for a system corresponding to linearly independent vectors?

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9. What can we say about the pivots for the augmented matrix for a system corresponding to linearly independent vectors?

If a set of vectors is not l.i. then at least one c_i is nonzero, say it is c_1, and then v_1 is a linear combination of the other vectors. If a set is l.i. then no vector is redundant, as in throwing any away would span a smaller space, and so the full set is an efficient set in this sense.

10. Evaluate the following statements:

a) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ is linearly independent

b) span of $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ is $\mathbb{R}^2$

c) both a) and b)

d) neither

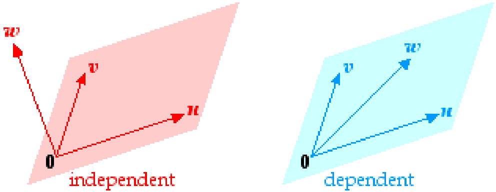


Image Credit: Min Yan

http://algebra.math.ust.hk/vector_space/07_independence/lecture2.shtml

11. Evaluate the following statements:

a) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$ is linearly independent

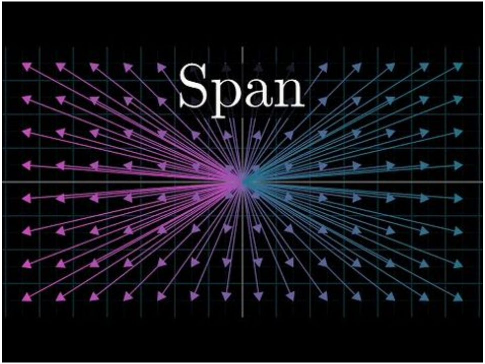
b) span of $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$ is $\mathbb{R}^2$

c) both a) and b)

d) neither

12. In the hw from 1.4, in #13, the problem asked whether $\vec{u}$ was in the plane spanned by the columns of A. The answer is...

a) yes and I have a good reason why
b) yes but I am not sure why
c) no but I am not sure why not
d) no and I have a good reason why not
e) what's a "span"?



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13. In Problem Set 1 number 1, the set of solutions is?

a) a point
b) a line
c) does not exist
d) a hyperplane in higher dimensions
e) non-linear

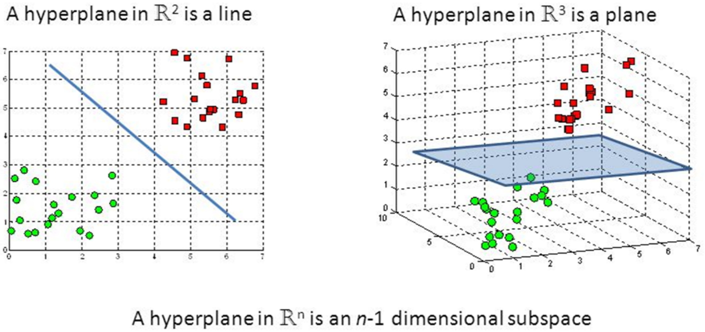
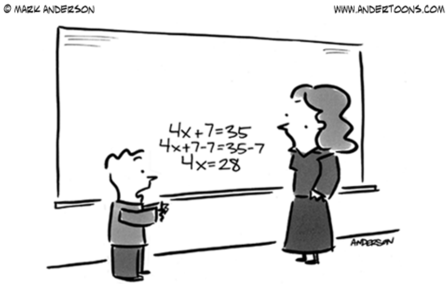


Image Source: Ankit Dixit *Ensemble Machine Learning: A beginner's guide that combines powerful machine learning algorithms to build optimized models*

14. A linear system has how many solutions?

a) 0 or 1
b) 0 or infinite
c) 0, 1 or infinite
d) 0, 1, 2 or infinite
e) none of the above

School Cartoon #6446



"You *knew* X was 7 the *whole time* and you *never said anything*?!"

http://www.mathplane.com/yahoo_site_admin/assets/images/mark_anderson_school_cartoon_6446_algebra.6574705_std.png

15. A homogeneous linear-system has how many solutions?

a) 0 or 1
b) 0 or infinite
c) 0, 1 or infinite
d) 0, 1, 2 or infinite
e) none of the above

OED: *Origin Early 17th century (as homogeneity): from medieval Latin homogeneus, from Greek homogenes, from homos 'same' + genos 'race, kind'.*

16. $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ span

a) a point

b) a line

c) does not exist

d) a hyperplane in higher dimensions

e) non-linear

Earliest Known Uses of some of the Words of Mathematics: *LINEAR COMBINATION occurs in "On the Extension of Delaunay's Method in the Lunar Theory to the General Problem of Planetary Motion," G. W. Hill, Transactions of the American Mathematical Society, Vol. 1, No. 2. (Apr., 1900).*

17. Evaluate the statement: The columns of an $m \times n$ coefficient matrix span $\mathbb{R}^m$ exactly when the augmented matrix reduces to one with a pivot for each column except the equals column

a) True and I can explain why
b) True but I am unsure of why
c) False but I am unsure of why not
d) False and I can give a counterexample

Except for boolean algebra there is no theory more universally employed in mathematics than linear algebra [Jean Dieudonné]

18. To check whether a vector is in the span of other vectors, it suffices to see if they are multiples

a) True and I can explain why
b) True but I am unsure of why
c) False but I am unsure of why not
d) False and I can give a counterexample

You can't learn too much linear algebra [Benedict Gross]

19. Evaluate the statement: If a collection of vectors is *not* l.i. then we could throw away *any* one vector and still span the same space

a) True and I can explain why
b) True but I am unsure of why
c) False but I am unsure of why not
d) False and I can give a counterexample

20. Which set of vectors is linearly independent?

a) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$

c) $\begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix}$

d) None of these sets are linearly independent.

e) Exactly two of these sets are linearly independent.

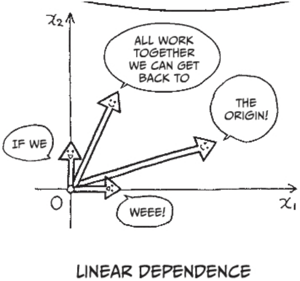


Image citation: Shin Takahashi and Iroha Inoue *The Manga Guide to Linear Algebra*

21. Suppose that the augmented matrix for a system reduces to $\begin{bmatrix} 1 & -4 & 2 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$. Describe the solutions, the intersections of the rows, geometrically and in parametric vector form.

a) a line $t \begin{bmatrix} 1 \\ -4 \\ 2 \\ 3 \end{bmatrix}$ in $\mathbb{R}^4$

b) a plane $s \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ in $\mathbb{R}^3$

c) another line

d) another plane

e) none of the above