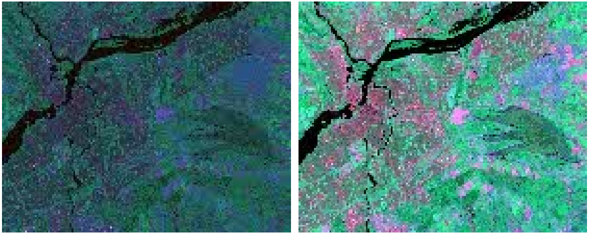
1. Define $T(\vec{v}) = A\vec{v}$, where A is $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Then $T(\vec{v})$

a) reflects $\vec{v}$ about the y-axis
b) reflects $\vec{v}$ about the x-axis
c) rotates $\vec{v}$ clockwise $\frac{\pi}{2}$ radians about the origin
d) rotates $\vec{v}$ counterclockwise $\frac{\pi}{2}$ radians about the origin
e) none of the above

Math isn't important for programming and other hilarious jokes you can tell yourself

2. Define $T(\vec{v}) = A\vec{v}$, where A is $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Then $T(\vec{v})$

a) reflects $\vec{v}$ about the y-axis
b) reflects $\vec{v}$ about the x-axis
c) rotates $\vec{v}$ clockwise $\frac{\pi}{2}$ radians about the origin
d) rotates $\vec{v}$ counterclockwise $\frac{\pi}{2}$ radians about the origin
e) none of the above



Before and after linear stretch

Image credit: https://www.nrcan.gc.ca/earth-sciences/geomatics/satellite-imagery-air-photos/satellite-imagery-products/educational-resources/9389

3. Define $T(\vec{v}) = A\vec{v}$, where A is $\begin{bmatrix} .5 & .5 \\ .5 & .5 \end{bmatrix}$. Then the range of T, the set of outputs of the linear transformation, is

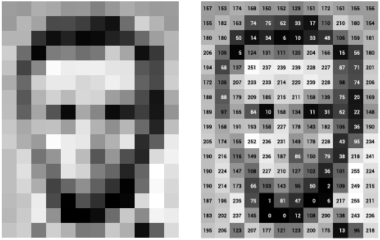
a) $\mathbb{R}^2$ and I have a good reason why

b) $\mathbb{R}^2$ but I am unsure of why

c) the $y = x$ line but I am unsure of why

d) the $y = x$ line and I have a good reason why

e) other



157	153	174	168	150	152	129	151	172	161	155	156
155	182	163	74	75	62	33	17	110	210	180	154
180	180	50	14	34	6	10	33	48	106	159	181
206	109	5	124	131	111	120	204	166	15	56	180
194	68	137	251	237	239	239	228	227	87	71	201
172	105	207	233	233	214	220	239	228	98	74	206
188	88	179	209	185	215	211	158	139	75	20	169
189	97	165	84	10	168	134	11	31	62	22	148
199	168	191	193	158	227	178	143	182	106	36	190
205	174	155	252	236	231	149	178	228	43	95	234
190	216	116	149	236	187	86	150	79	38	218	241
190	224	147	108	227	210	127	102	36	101	255	224
190	214	173	66	103	143	96	50	2	109	249	215
187	196	235	75	1	81	47	0	6	217	255	211
183	202	237	145	0	0	12	108	200	138	243	236
195	206	123	207	177	121	123	200	175	13	96	218

Image credit: Zach Lieberman

https://openframeworks.cc/ofBook/chapters/image_processing_computer_vision.html

4. To rotate a figure $\begin{bmatrix} x_1 & \dots & x_p \\ y_1 & \dots & y_p \\ 1 & \dots & 1 \end{bmatrix}$ about a point (-2,3) we can:

a)
$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 & \dots & x_p \\ y_1 & \dots & y_p \\ 1 & \dots & 1 \end{bmatrix}$$

b)
$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 & \dots & x_p \\ y_1 & \dots & y_p \\ 1 & \dots & 1 \end{bmatrix}$$

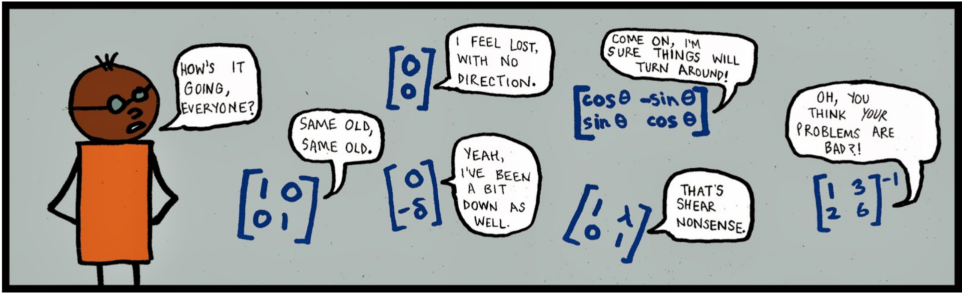
c) $\begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 & \dots & x_p \\ y_1 & \dots & y_p \\ 1 & \dots & 1 \end{bmatrix}$

d) more than one of the above

e) none of the above

5. Which of the following are true about linear transformations?

a) points go in as column vectors, and the first column of the transformation represents the output of the unit *x*-axis
b) we must use homogeneous coordinates (higher dimensional coordinate=1) if we want to use translations
c) they compose like functions with the first transformation on the right
d) two of the above
e) all of a), b) and c)



https://pbs.twimg.com/media/BhKmT5RIYAA4iU6.jpg:large

6. Let $T : \vec{x} \rightarrow A\vec{x}$ be given as a linear transformation arising from a square 2x2 matrix A. Assume that the set of all outputs $\vec{b}$ (from $A\vec{x} = \vec{b}$) is a line. What can we deduce?

a) The columns of A do not span $\mathbb{R}^2$ and I can think of an example
b) The columns of A do not span $\mathbb{R}^2$ but I can not think of an example
c) The columns of A span $\mathbb{R}^2$ and I can think of an example
d) The columns of A span $\mathbb{R}^2$ but I can not think of an example

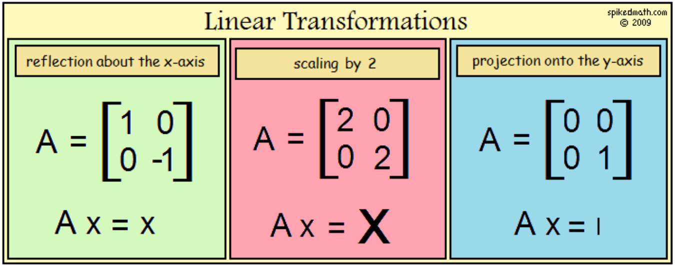
6. Let $T : \vec{x} \to A\vec{x}$ be given as a linear transformation arising from a square 2x2 matrix A. Assume that the set of all outputs $\vec{b}$ (from $A\vec{x} = \vec{b}$) is a line. What can we deduce?

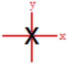
a) The columns of A do not span $\mathbb{R}^2$ and I can think of an example

b) The columns of A do not span $\mathbb{R}^2$ but I can not think of an example

c) The columns of A span $\mathbb{R}^2$ and I can think of an example

d) The columns of A span $\mathbb{R}^2$ but I can not think of an example





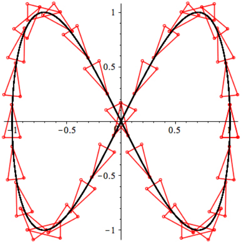
http://spikedmath.com/comics/109-linear-transformations.png

7. To turn a car so that it points in the direction of motion, we

a) define a unit vector in the direction of the velocity (tangent) of the curve by dividing by its length/norm so it won't change the size of the car

b) create an orthogonal vector to pair with it in a rotation matrix by creating a vector on a line with negative reciprocal slope (swap *x* and *y* and introduce a negative sign)

c) both of the above



VLA Package from *Visual Linear Algebra* by Herman and Pepe

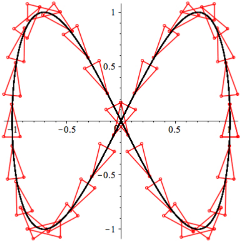
8. To keep a car on a curved race track, we can perform the appropriate matrix operations in the following order

a) $\begin{bmatrix} 1 & 0 & \cos(t) \\ 0 & 1 & \sin(2t) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{-\sin(t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & \frac{-2\cos(2t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & 0 \\ \frac{-2\cos(2t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & \frac{-\sin(t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ car

b) $\begin{bmatrix} \frac{-\sin(t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & \frac{-2\cos(2t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & 0 \\ \frac{-2\cos(2t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & \frac{-\sin(t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \cos(t) \\ 0 & 1 & \sin(2t) \\ 0 & 0 & 1 \end{bmatrix}$ car

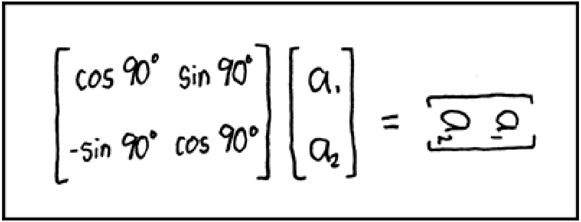
c) either works

d) none of the above



9. To rotate Yoda, who was given in row vectors as opposed to column vectors, we made use of

a) a matrix that rotates about a line in 3-space
b) $(AB)^T = B^T A^T$
c) Both of the above
d) none of the above



https://xkcd.com/184/