1. What size is this matrix? Matrix([[6,11,-2],[23,31,5]])
(a) $2 \times 3$
() $3 \times 2$
(2) 6

2. Let $A=\left[\begin{array}{cc}4 & 6 \\ 20 & 7\end{array}\right]$. What is $5 A$ ?
(a) $\left[\begin{array}{cc}9 & 6 \\ 20 & 7\end{array}\right]$
(2) $\left[\begin{array}{cc}9 & 11 \\ 25 & 12\end{array}\right]$
(2) $\left[\begin{array}{ll}20 & 6 \\ 20 & 7\end{array}\right]$
(1) $\left[\begin{array}{cc}20 & 30 \\ 100 & 35\end{array}\right]$


Before and after linear stretch

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Image credit: https://www.nrcan.gc.ca/earth-sciences/geomatics/
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satellite-imagery-air-photos/satellite-imagery-products/educational-resources/9389
3. Let $A=\left[\begin{array}{cc}4 & 6 \\ 20 & 24\end{array}\right]$ and $B=\left[\begin{array}{ll}2 & 5 \\ 3 & 7\end{array}\right]$ What is $A+B$ ?
(a) 71
(2) $\left[\begin{array}{cc}6 & 9 \\ 7 & 11\end{array}\right]$
(a) $\left[\begin{array}{cc}6 & 11 \\ 23 & 31\end{array}\right]$
(1) $\left[\begin{array}{cccc}4 & 6 & 2 & 5 \\ 20 & 24 & 3 & 7\end{array}\right]$


Image credit: Zach Lieberman
https://openframeworks.cc/ofBook/chapters/image_processing_computer_vision.html
4. A fruit grower raises two crops, which are shipped to three outlets.

The number of units of product $i$ that is shipped to outlet $j$ is
represented by $b_{i j}$ in the matrix $B=\left[\begin{array}{ccc}100 & 75 & 75 \\ 125 & 150 & 100\end{array}\right]$
The profit of one unit of product $i$ is represented by $a_{1 i}$ in the matrix $A=\left[\begin{array}{ll}\$ 3.75 & \$ 7.00\end{array}\right]$

Does the matrix multiplication $B A$ make sense?
(a) yes and I have a good reason why
(0) yes but I am unsure of why
(0) no but I am unsure of why not
(1) no and I have a good reason of why not
4. A fruit grower raises two crops, which are shipped to three outlets.

The number of units of product $i$ that is shipped to outlet $j$ is
represented by $b_{i j}$ in the matrix $B=\left[\begin{array}{ccc}100 & 75 & 75 \\ 125 & 150 & 100\end{array}\right]$
The profit of one unit of product $i$ is represented by $a_{1 i}$ in the matrix $A=\left[\begin{array}{ll}\$ 3.75 & \$ 7.00\end{array}\right]$

Does the matrix multiplication $B A$ make sense?
(1) yes and I have a good reason why
(0) yes but I am unsure of why
(2) no but I am unsure of why not
(1) no and I have a good reason of why not How about $A B$ ?
Does any entry look like $3.75 \times 100+3.75 \times 125 ?$
5. There exists a matrix $A=\left[\begin{array}{ll}a & b \\ c & d\end{array}\right]$ so that $\left[\begin{array}{ll}1 & 2 \\ 1 & 2\end{array}\right] A=\left[\begin{array}{ll}1 & 1 \\ 1 & 1\end{array}\right]$
(0) there is exactly 1 matrix $A$ that works
(0) there are infinitely many matrices $A$ that work
(0) there are no matrices that work
(0) none of the above


Image credit: Guy vandegrift
https://commons.wikimedia.org/wiki/File:Hands_matrix_multiplication.svg
6. What is true about elementary matrices?
(1) $\left.\begin{array}{lll}3 & 1 & 0\end{array}\right]$ is the same as modifying $A$ via $r_{2}^{\prime}=3 r_{1}+r_{2}$
(0) To find an elementary matrix we can apply the row operation to I
(0 $A^{-1}$ is the product of the elementary matrices that reduce $A$ to $/\left[E_{p} \ldots E_{2} E_{1}\right]$
(0) all of the above

http://www.mathplane.com/gate_dwebcomics/math_comics_archiye_fall_2013
7. There exists a matrix $A=\left[\begin{array}{ll}a & b \\ c & d\end{array}\right]$ so that $A\left[\begin{array}{ll}1 & 1 \\ 1 & 1\end{array}\right]=\left[\begin{array}{ll}1 & 2 \\ 1 & 2\end{array}\right]$
(2) there is exactly 1 matrix $A$ that works
(0) there are infinitely many matrices $A$ that work
(0) there are no matrices that work

8. If $A=\left[\begin{array}{ccc}2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4\end{array}\right]$ then what is $A^{T}$ ?
(a) $\left[\begin{array}{ccc}2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4\end{array}\right]$
(2) $\left[\begin{array}{ccc}2 & 0 & -2 \\ 3 & -1 & 0 \\ 1 & 3 & 4\end{array}\right]$
(3) $\left[\begin{array}{ccc}-2 & 0 & 4 \\ 0 & -1 & 3 \\ 2 & 3 & 1\end{array}\right]$
(1) $\left[\begin{array}{ccc}1 & 3 & 4 \\ 3 & -1 & 0 \\ 2 & 0 & -2\end{array}\right]$
(2) none of the above
9. You have a business that sells tables and chairs. You have brown tables and white tables, and corresponding chairs. Your May sales are 4 brown tables, 6 white tables, 20 brown chairs, and 24 white chairs, which is represented by the matrix $M=\left[\begin{array}{cc}4 & 6 \\ 20 & 24\end{array}\right]$ where the first row is tables, the second row is chairs, the first column is brown items, and the second column is white items. Your June sales are given by the matrix
$J=\left[\begin{array}{cc}6 & 8 \\ 22 & 32\end{array}\right]$. What matrix operations would make sense in real life in this scenario? Be prepared to discuss why or why not for each.
(0) $M+J$
(1) $M-J$
(-1.2J
© $M J$
(-) More than one of the above makes sense
10. In the homework due today, did you use something similar to the following critical analysis/reasoning?
Multiply both sides of an equation by the inverse of a matrix
Reorder parenthesis by associativity to pair a matrix with its inverse
Cancel A by its inverse: $A^{-1} A=I_{n \times n}$ or $A^{-1} A=I_{n \times n}$
Identity reduces
(1) Yes and I used it more than once
(- Yes and I used it once

- No, although I used some of this reasoning
- No, I didn't use anything like it
- What homework?


11. If $A$ is an invertible $n \times n$ matrix, and $\vec{x}$ and $\vec{b}$ are $n \times 1$ vectors, then the matrix-vector equation $A \vec{x}=\vec{b}$ has a unique solution
(2) True and I can tell you what the solution is
(0) True but I am unsure what the solution is
(2) Always false
(1) False but holds at times

Silliness: Who writes inverse?
11. If $A$ is an invertible $n \times n$ matrix, and $\vec{x}$ and $\vec{b}$ are $n \times 1$ vectors, then the matrix-vector equation $A \vec{x}=\vec{b}$ has a unique solution
(2) True and I can tell you what the solution is
(0) True but I am unsure what the solution is
(2) Always false
(1) False but holds at times

Silliness: Who writes inverse?
A backwards poet.
12. Evaluate the statement: If $A \vec{x}=\overrightarrow{0}$, then is $C(A \vec{x})=\overrightarrow{0} C$ ?
(0) Yes and I have a good reason why
(0) Yes but I am unsure of why
( - No but I am unsure of why not
(0) No and I have a good reason why not

http://spikedmath.com/131.html
13. If $A$ is an invertible $n \times n$ matrix, with $n>1$, and $\vec{x}$ and $\vec{b}$ are $1 \times n$ vectors, then the matrix-vector equation $A \vec{x}=\vec{b}$ has a unique solution
(2) True and I can tell you what the solution is
(a) True but I am unsure what the solution is
(2) Always false
(1) False but holds at times

http://mrburkemath.net/xwhy/images/776-5x501.jpg
14. If $A$ is not invertible and $A B=A C$, must $B=C$ ?
(2) Yes and I have a good reason why
(0) Yes but I am unsure of why
(-) No but I am unsure of why not
© No and I have a counterexample

# MATHEMATICS <br> <br> $\overline{\overline{\text { is }}}$ not $a \overline{\overline{\text { I }}}$ <br> <br> $\overline{\overline{\text { is }}}$ not $a \overline{\overline{\text { I }}}$ SPECTATOR SPORT 

15. Given $A_{n \times n}$ (square), can $A \vec{x}=\overrightarrow{0}$ have only the trivial solution?
(2) no that statement is impossible
(2) yes when the columns of $A$ are l.i. but we can't say anything more
(2) yes when the columns of $A$ are I.i. and A has $n$ pivot rows
(1) yes when the columns of $A$ are I.i. and $A$ has $n$ pivot columns
() both c and d

In linear algebra, the trivial solution is the $\overrightarrow{0}$. Nontrivial solutions are additional solutions for homogeneous systems, if they exist. The definition of linearly independent also makes use of this language. The concept first appears in 1.5 in the book.
16. Given $A_{m \times n}$ (not square), can $A \vec{x}=\overrightarrow{0}$ have only the trivial solution?
(D) no that statement is impossible
(0) yes when the columns of $A$ are I.i. but we can't say anything more
(0) yes when the columns of $A$ are I.i. and $A$ has $n$ pivot rows
(0) yes when the columns of $A$ are I.i. and $A$ has $n$ pivot columns
(-) both c and d

In linear algebra, the trivial solution is the $\overrightarrow{0}$. Nontrivial solutions are additional solutions for homogeneous systems, if they exist. The definition of linearly independent also makes use of this language. The concept first appears in 1.5 in the book.
17. For the Hill Cipher
(2) $A_{n \times n}$ [original message $]_{n \times p}=[\text { coded message }]_{n \times p}$
(0) to decode, we must use apply an invertible matrix to the coded message and read the message along the rows
(0) the method is vulnerable to those that intercept enough coded/decoded vector correspondances because of its linearity
(1) all of the above
(2) two of the above


Image Credit: 1932 Patent Application 1,845,947 https://patents.google çom/patent/US1845947A/en
18. If the condition number of a square matrix with fractional entries is $3.5 \times 10^{6}$ then...
(a) we should use 8 decimal places in our measurements of $\vec{b}$ if we want solutions to $A \vec{x}=\vec{b}$ to be accurate to 2 decimal places
(0) the matrix is invertible
(3) both of the above
() none of the above

Try it Out!
Practice
Computations
and Critical
Analysis

## Apply Linear Algebra to <br> Numerous <br> Situations

19. The equation $A \vec{x}=\vec{b}$ has at least one solution for each $\vec{b}$ in $\mathbb{R}^{n}$ whenever $A$ is an $n \times n$ matrix.
( - true
(0) false and I can think of a counterexample
(0) false and I can think of a correction
(©) both b) and c)
(0) other

http://www.mathfunny.com/images/
20. If there is a $\vec{b}$ in $\mathbb{R}^{n}$ such that the equation $A \vec{x}=\vec{b}$ is consistent, where $A$ is $n \times n$, then the solution is unique.
(c) true
(0) false and I can think of a counterexample
(-) false and I can think of a correction
(0. both b) and c)
(0) other

21. If the columns of a $7 \times 7$ matrix $D$ are linearly independent, what can be said about the solutions $D \vec{x}=\vec{b}$ for a given $7 \times 1$ vector $\vec{b}$ (where $\vec{x}$ is $7 \times 1$ too)?
(a) $D \vec{x}=\vec{b}$ always has at least one solution, but we cannot say anything more about the solution or solutions
(D) $D \vec{x}=\vec{b}$ always has a unique solution, but we cannot say anything more about it
(0) $D \vec{x}=\vec{b}$ always has a unique solution, and I can tell you what it is
(1) $D \vec{x}=\vec{b}$ always has infinite solutions
() $D \vec{x}=\vec{b}$ has no solutions for some $\vec{b}$ and infinite solutions for other $\vec{b}$
22. If the columns of a $7 \times 6$ matrix $D$ are linearly independent, what can be said about the solutions $D \vec{x}=\vec{b}$ for a given $7 \times 1$ vector $\vec{b}$ (where $\vec{x}$ is $6 \times 1$ )
(a) $D \vec{x}=\vec{b}$ always has at least one solution, but we cannot say anything more about the solution or solutions
(0) $D \vec{x}=\vec{b}$ always has a unique solution
() $D \vec{x}=\vec{b}$ has no solutions for some $\vec{b}$ and infinite solutions for other $\vec{b}$
(1) $D \vec{x}=\vec{b}$ has one solution for some $\vec{b}$ and no solutions for other $\vec{b}$
(2) We can reason that $D \vec{x}=\vec{b}$ has 0,1 , or infinite solutions as with any linear system, but we cannot specify the solutions any further.
