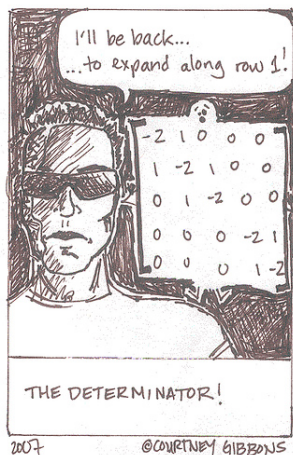


a) Number of terms in the determinant

$2 \times 2$  matrix has 2 terms in the determinant,  $3 \times 3$  has 6 terms, and  $4 \times 4$  has 24 terms. Do you see a pattern to the number of terms? How many terms will an  $n \times n$  matrix have in its determinant? Hint: something from Calculus 2 and Analytic Geometry is useful here!

b) Compute the determinant as directed



<http://brownsharpie.courtneygibbons.org/comic/determinator/>

$$\sum_1^n a_{ij} \cdot (-1)^{i+j} \cdot \text{Det of matrix obtained by eliminating row } i \text{ and column } j \text{ where } i = 1 \text{ is fixed, } j = 1..5$$

c) Compute the matrices and their determinants and compare. Does the determinant change? If so, how is it related to the original? Also, what kind of linear transformations are the first three?

1)  $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$  versus determinant of matrix after  $r'_2 = -3r_1 + r_2$

2)  $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$  versus determinant of matrix after  $r_1 \leftrightarrow r_2$

3)  $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$  versus determinant of the matrix after  $r'_2 = cr_2, c \neq 0$

4)  $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$  versus determinant of the transpose of the matrix

d) What is the determinant of a triangular matrix like  $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 8 & 9 \\ 0 & 0 & 0 & 10 \end{vmatrix}$  or  $\begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & 5 & 0 & 0 \\ 3 & 6 & 8 & 0 \\ 4 & 7 & 9 & 10 \end{vmatrix}$ ?

e) What is the determinant of the inverse of a matrix?

- write the inverse of  $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ . What is its determinant and how does it compare to the original?
- via matrix algebra
- via elementary row operations