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`Eigenvalues(A);`

and obtain the output $h, P :=$ $\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix}, \begin{bmatrix} | & | & \dots \\ v_1 & v_2 & \dots \\ | & | & \dots \end{bmatrix}$

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D=diagonal matrix with eigenvalues on the diagonal

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If the eigenvectors form a basis for \mathbb{R}^n then P is invertible and

$P^{-1}AP = D$, i.e. A is called *diagonalizable*. Equivalently

$$A = PDP^{-1}$$

Application to computer graphics: Execute in Maple:

```
Proj := Matrix([[(cos(theta))^2, cos(theta) *  
sin(theta)], [cos(theta) * sin(theta), ((sin(theta))^2)]]);  
h,P:=Eigenvectors(Proj);  
Diag:=simplify(MatrixInverse(P).A.P);
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1. What geometric transformation is Diag?

- a) rotation
- b) reflection
- c) shear
- d) projection
- e) dilation

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Writing out a transformation $A = PDP^{-1}$ is very useful in computer graphics: If we want to project a vector onto the $y = \tan(\theta)x$ line, first P^{-1} takes a vector and rotates it clockwise by theta [to the x-axis]. Next we perform Diag, which projects onto the x-axis, and finally we perform P, which rotates counterclockwise by theta [back to where we began].