1. In Maple we execute

A := Matrix([[21/40,3/20],[-3/16,39/40]]);

Eigenvectors(A);

and obtain the output $\begin{bmatrix} \frac{3}{5} \\ \frac{9}{10} \end{bmatrix}, \begin{bmatrix} 2 & \frac{2}{5} \\ 1 & 1 \end{bmatrix}$ Notice that the eigenvectors of $\begin{bmatrix} \frac{21}{40} & \frac{3}{20} \\ -\frac{3}{16} & \frac{39}{40} \end{bmatrix}$ span $\mathbb{R}^2$.

Using the Maple output, write out the eigenvector decomposition.

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Using the Maple output, write out the eigenvector decomposition.

What does the trajectory look like for an initial vector in quadrant 1 that does not begin on either eigenvector?

a) dies off to the origin asymptotic to one eigenvector (dominant eigenvalue < 1)

b) grows asymptotic to one eigenvector (dominant eigenvalue $>$1)

c) comes in parallel to one eigenvector with smaller and smaller contributions until we hit the other (dominant eigenvalue = 1)

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Say that A represents the changes from one year to the next in a system of foxes (x-value) and rabbits (y-value). For most initial conditions, what happens to the system in the longterm?

a) populations die off in the ratios of 2 foxes to 1 rabbit

b) populations die off in the ratios of 1 fox to 2 rabbits

c) populations die off in the ratios of 2 foxes to 5 rabbits

d) populations die off in the ratios of 5 foxes to 2 rabbits

e) other longterm behavior

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d) populations die off in the ratios of 5 foxes to 2 rabbits
e) other longterm behavior

For most initial conditions, the rate of die off in the longterm is 10% each year = 1 - dominant eigenvalue.
Sketch a trajectory diagram

3. Given a square matrix A, to solve $A\vec{x} = \lambda\vec{x}$ for eigenvalues and eigenvectors

a) $(A - \lambda I)\vec{x} = \vec{0}$ is equivalent, so, since we are looking for nontrivial $\vec{x}$ solutions, that means that this homogeneous system must have infinite solutions, so we can solve for $\det(A - \lambda I) = -1$

b) Once we have a λ that works, we can take the inverse of $A - \lambda I$ to solve for the eigenvectors

c) Once we have a λ that works, we can create the augmented matrix $[A - \lambda I \;\; \vec{0}]$ and reduce to solve for the nullspace of $A - \lambda I$ (eigenspace of A), and write out a basis

4. A reflection matrix has eigenvalue(s)

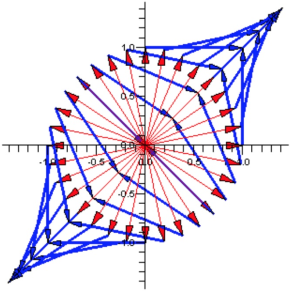
a) $\lambda = 1$ on the line of reflection

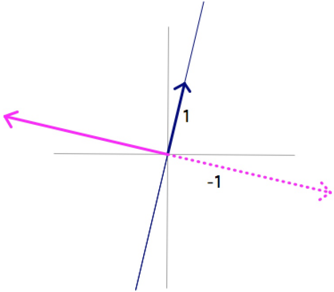
b) $\lambda = -1$ perpendicular to the line of reflection

c) $\lambda = -2$ for some line

d) all of the above

e) two of the above





5. An eigenvector $\vec{x}$ allows us to turn matrix

a) multiplication into matrix addition
b) addition into matrix multiplication
c) multiplication into scalar multiplication
d) addition into scalar multiplication
e) none of the above

Euler: principal axes for the rotational motion of a rigid body.
Lagrange: they are the eigenvectors of the inertia matrix
Cauchy: generalized [Hawkins, 1975]
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$A\vec{x} = \lambda\vec{x}$

In the original matrix p is called a *predation parameter*:

$\begin{bmatrix} \frac{21}{40} & \frac{3}{20} \\ -p & \frac{39}{40} \end{bmatrix}$. Find a value of p so that the populations tend towards constant levels (stability).

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plug in $\lambda = 1$:
0 = determinant $(A - \lambda I) =$ determinant $(A - 1 \cdot I)$
$= \det\left(\begin{bmatrix} \frac{21}{40} & \frac{3}{20} \\ -p & \frac{39}{40} \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{vmatrix} \frac{21}{40} - 1 & \frac{3}{20} \\ -p & \frac{39}{40} - 1 \end{vmatrix} = \frac{19}{1600} + \frac{3}{20}p$
$p = -\frac{19}{240}$

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A := Matrix([[21/40,3/20],[19/240,39/40]]);

Eigenvectors(A); $\begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}, \begin{bmatrix} -6 & \frac{6}{19} \\ 1 & 1 \end{bmatrix}$

6. A := Matrix([[21/40,3/20],[19/240,39/40]]);
Eigenvectors(A); $\begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}, \begin{bmatrix} -6 & \frac{6}{19} \\ 1 & 1 \end{bmatrix}$

What are the relative population sizes in the longterm? [foxes (x-value) and rabbits (y-value)]

a) -6 foxes to 1 rabbit
b) 1 fox to -6 rabbits
c) 6 foxes to 19 rabbits
d) 19 foxes to 6 rabbits
e) none of the above

7. A := Matrix([[21/40,3/20],[19/240,39/40]]);

Eigenvectors(A); $\begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}, \begin{bmatrix} -6 & \frac{6}{19} \\ 1 & 1 \end{bmatrix}$

The eigenvector decomposition:

$$\begin{bmatrix} \text{Foxes}_k \\ \text{Rabbits}_k \end{bmatrix} = \left(\frac{1}{2}\right)^k \begin{bmatrix} -6a_1 \\ a_1 \end{bmatrix} + \begin{bmatrix} \frac{6}{19}a_2 \\ a_2 \end{bmatrix}$$

What does the trajectory look like for an initial vector in quadrant 1 that does not begin on either eigenvector?

a) approaches the origin asymptotic to one eigenvector
b) comes in parallel to one eigenvector with smaller and smaller contributions until we hit the other.
c) grows asymptotic to one eigenvector
d) more than one of the above
e) none of a, b, c

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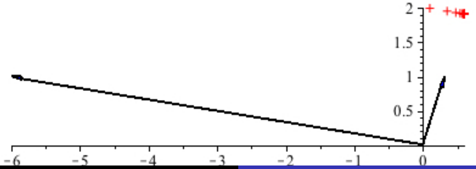

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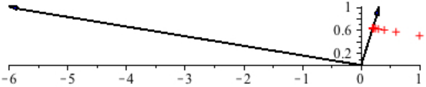

8. The eigenvector decomposition for a system of owls (x-value) and wood rats (y-value) is given via

$$\vec{x}_k = \begin{bmatrix} \text{owls}_k \\ \text{wood rats}_k \end{bmatrix} = a_1\left(\frac{7}{10}\right)^k \begin{bmatrix} 2 \\ 1 \end{bmatrix} + a_2\left(\frac{9}{10}\right)^k \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Which of the following is true about the populations in the longterm for most starting positions?

a) the owls (x-value) dominate

b) the wood rats (y-value) dominate

c) the owl population crashes faster than the wood rats

d) more than one of the above

e) none of the above

9. $\vec{x_k} = \begin{bmatrix} \text{owls}_k \\ \text{wood rats}_k \end{bmatrix} = a_1(\frac{7}{10})^k \begin{bmatrix} 2 \\ 1 \end{bmatrix} + a_2(\frac{9}{10})^k \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Which of the following is true about the trajectory of $\vec{x}_k$ for most starting positions?

a) the owls die off along $y = x$ but the wood rats don't
b) the wood rats die off along $y = x$ but the owls don't
c) the owls and wood rats die off in the ratio of 2 owls to 1 wood rat along $y = \frac{1}{2}x$
d) the owls and wood rats die off in the ratio of 1 owl to 1 wood rat along y=x
e) none of the above

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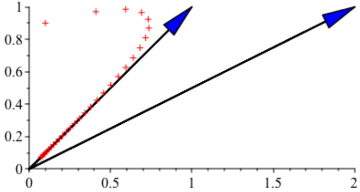

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When do we die off along $y = \frac{1}{2}x$

a) never
b) always
c) for most starting positions
d) only when $a_1 = 0$
e) only when $a_2 = 0$

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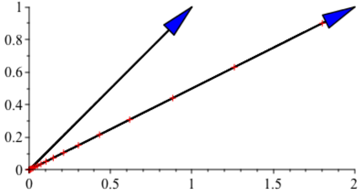

11. Could we write out an eigenvector decomposition for a reflection matrix (i.e. are there 2 linearly independent eigenvectors that span $\mathbb{R}^2$)

a) yes and I can tell you how the eigenvectors relate to the line of reflection
b) yes but I am unsure of what they are
c) no but I am unsure of why not
d) no and I can explain why not

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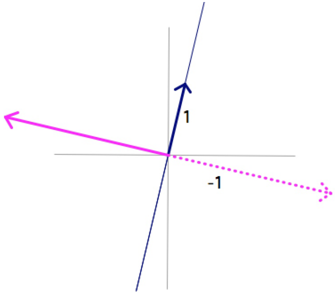
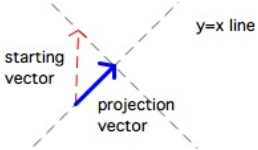


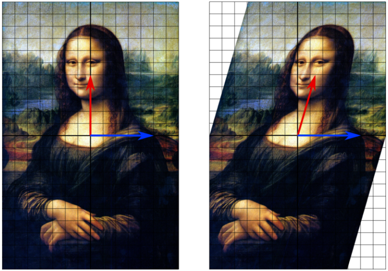
12. Could we write out an eigenvector decomposition for a projection matrix (i.e. are there 2 linearly independent eigenvectors that span $\mathbb{R}^2$)

a) yes and I can tell you how the eigenvectors relate to the line of projection
b) yes but I am unsure of what they are
c) no but I am unsure of why not
d) no and I can explain why not



13. Could we write out an eigenvector decomposition for a horizontal shear matrix (i.e. are there 2 linearly independent eigenvectors that span $\mathbb{R}^2$)

a) yes and I can tell you how the eigenvectors relate to the horizontal shear
b) yes but I am unsure of what they are
c) no but I am unsure of why not
d) no and I can explain why not



TreyGreer62 CC0

14. Which of the following must be true if $A_{2\times 2}$ has a nonzero eigenvector $\vec{x}$ satisfying $A\vec{x} = 5\vec{x}$?

a) A's eigenvectors are all of $\mathbb{R}^2$
b) A must have at least an entire line through the origin in $\mathbb{R}^2$ as its eigenvectors, where the vectors get stretched by 5
c) A can have just the one eigenvector $\vec{x}$ that is stretched by 5
d) A has exactly two eigenvectors, the second being from Maple
e) none of the above

14. Which of the following must be true if $A_{2\times 2}$ has a nonzero eigenvector $\vec{x}$ satisfying $A\vec{x} = 5\vec{x}$?

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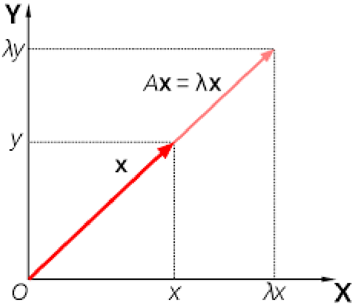

Lyudmil Antonov Lantonov CC4

15. If the reduced augmented matrix for the system $(A - \lambda I)\vec{x} = \vec{0}$ is $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ with A as a 2×2 matrix then the non-zero (real) eigenvectors of A are

a) none
b) a line through the origin
c) all of $\mathbb{R}^2$
d) a subspace of $\mathbb{R}^3$ (with 3 coordinates)
e) none of the above

16. What are the non-zero real eigenvectors of $\begin{bmatrix} \cos\pi & -\sin\pi \\ \sin\pi & \cos\pi \end{bmatrix}$?

a) none

b) a line of eigenvectors

c) two different lines of eigenvectors

d) all of $\mathbb{R}^2$

e) none of the above

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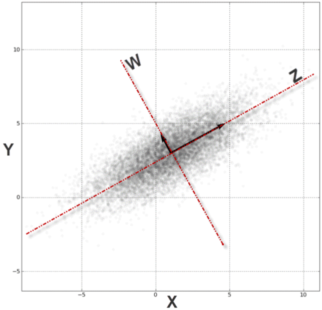
17. If we write a **basis** for the eigenspace of $\begin{bmatrix} \cos\pi & -\sin\pi \\ \sin\pi & \cos\pi \end{bmatrix}$, how many vectors does it have?

a) 0
b) 1
c) 2
d) infinite
e) none of the above



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18. What are the non-zero real eigenvectors of $\begin{bmatrix} \cos\frac{\pi}{5} & -\sin\frac{\pi}{5} \\ \sin\frac{\pi}{5} & \cos\frac{\pi}{5} \end{bmatrix}$

a) none
b) a line of eigenvectors
c) two different lines of eigenvectors
d) all of $\mathbb{R}^2$
e) none of the above