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- In the longterm, for most starting positions, the system (circle one): dies off, stabilizes, grows as the line with equation $y=$ $\qquad$ corresponding to the eigenvector $\qquad$ , except if the coefficient of $\qquad$ equals 0 , then the system (circle one): dies off, stabilizes, grows corresponding to $\qquad$ .

