Exam 1 *i-clicker* questions

Here are portions of questions to help you with your notes. The precise wording and ordering may change and we may not have time to cover all of these. The purpose of *i-clickers* is to actively practice concepts, computational strategies, critical & creative thinking, and communication. Making mistakes is integral to the learning process and enriches our understanding as we extend content and clear up misconceptions.

- **Think** about a possible answer(s) on your own.
- **Pair up**: discuss your thoughts in a group. We may reorganize groups at times.
- Prepare to **share** something from your group's discussion. This may take the form of an assertion, question, definition, example, or other connection. It could be something you tried and rejected.
- May be a lag at times—use this to **review** related concepts and examples, and **add** to your notes and the glossary, or get to know your neighbors.

Appalachian's General Education Program prepares students to employ various modes of communication. Successful communicators interact effectively with people of both similar and different experiences and values and in this class you will practice oral and written communication during class by interacting with various peers and me.

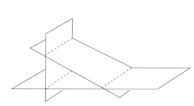
Clickers in 1.1 and 1.2

1. In 1.1 #19, the augmented matrix was
   Matrix([[1,h,4],[3,6,8]])
   Note this is Maple notation - each row of the matrix is in brackets. Eliminate the number 3 using Gaussian elimination...

2. What is the solution to the system of equations represented with this reduced augmented matrix $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$?

3. If a linear system with 3 equations and 3 variables is inconsistent then we must have...

4. How many solutions to a linear system of equations are possible?

5. According to the language of linear algebra, this picture  ...

6. How can we geometrically represent the parametric equations $(2t, -t+1, t)$?

7. For a system of three linear equations in three variables, which of the following scenarios would always guarantee an infinite number of solutions?

8. Use Gaussian on the following augmented matrix $\begin{bmatrix} 1 & 1 & 0 & 2 \\ 2 & 1 & 3 & 3 \\ 2 & 2 & h & 4 \end{bmatrix}$?

9. For full credit, what are the policies for graded problem sets?

Clickers in 1.3, 1.4, 1.5, and 1.7

1. Multiplying a column vector $\vec{v}_1$ by a real number c_1...

2. What do the collection of column vectors $c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, for c_1 and c_2 real, have in common?

3. Notice that $-1 \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} + 2 \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$. More generally, what do the collection of column vectors $c_1 \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$, for c_1 and c_2 real, have in common geometrically?

4. A coffee shop offers two blends of coffees: House and Deluxe. Each is a blend of Brazil, Colombia, Kenya, and Sumatra roasts:

	House	Deluxe
Brazil	30%	40%
Columbia	20%	30%
Kenya	20%	20%
Sumatra	30%	10%

. Suppose that the shop has 36 lbs of Brazil roast, 26 lbs of Columbia roast, 20 lbs of Kenya roast, and 18 lbs of Sumatra roast in stock and wants to completely use up the stock of coffee at hand in making the blends. What represents the corresponding system?

5. We perform the following in Maple:

```
s13n15extension:=Matrix([[1,-5,b1],[3,-8,b2],[-1,2,b3]]);
ReducedRowEchelonForm(s13n15extension);
```

and obtain the 3x3 identity $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$...

6. For two column vectors $\vec{v}_1$ and $\vec{v}_2$, $\{c_1\vec{v}_1 + \vec{v}_2$ so that c_1 is real$\}$ is...

7. Suppose that the augmented matrix for a system reduces to $\begin{bmatrix} 1 & -4 & 5 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Describe the solutions, the intersections of the rows, geometrically and in parametric vector form.

8. Compare the span of the 3 vectors $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, to the span of the 2 vectors $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

9. What can we say about the pivots for the augmented matrix for a system corresponding to linearly independent vectors?

10. Evaluate the following statements: $\{\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}\}$ is linearly independent

    span of $\{\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}\}$ is $\mathbb{R}^2$

11. Evaluate the following statements: $\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\}$ is linearly independent

    span of $\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\}$ is $\mathbb{R}^2$

12. In the hw from 1.4, in #13, the problem asked whether u was in the plane spanned by the columns of A. The answer is...

13. In Problem Set 1 number 1, the set of solutions is?

14. A linear-system has how many solutions?

15. A homogeneous linear-system has how many solutions?

16. $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}2\\2\end{bmatrix}$ span

17. Evaluate the statement: The columns of an $m \times n$ coefficient matrix span $\mathbb{R}^m$ exactly when the augmented matrix reduces to one with a pivot for each column except the equals column

18. To check whether a vector is in the span of other vectors, it suffices to see if they are multiples

19. Evaluate the statement: If a collection of vectors is not l.i. then we could throw away any one vector and still span the same space

20. Which set of vectors is linearly independent?

21. Suppose that the augmented matrix for a system reduces to $\begin{bmatrix}1 & -4 & 2 & 3\\0 & 0 & 1 & 1\end{bmatrix}$. Describe the solutions, the intersections of the rows, geometrically and in parametric vector form.

Clickers in 2.1, 2.2, and 2.3

1. What size is this matrix? Matrix([[6,11,-2],[23,31,5]])

2. Let $A = \begin{bmatrix}4 & 6\\20 & 7\end{bmatrix}$. What is $5A$?

3. Let $A = \begin{bmatrix}4 & 6\\20 & 24\end{bmatrix}$ and $B = \begin{bmatrix}2 & 5\\3 & 7\end{bmatrix}$ What is $A + B$?

4. A fruit grower raises two crops, which are shipped to three outlets. The number of units of product i that is shipped to outlet j is represented by b_{ij} in the matrix $B = \begin{bmatrix}100 & 75 & 75\\125 & 150 & 100\end{bmatrix}$. The profit of

one unit of product i is represented by a_{1i} in the matrix $A = \begin{bmatrix} \$3.75 & \$7.00 \end{bmatrix}$

Does the matrix multiplication BA make sense?

5. There exists a matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ so that $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

6. What is true about elementary matrices like $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$?

7. There exists a matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ so that $A \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$

8. If A= $\begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix}$ then what is A^T?

9. You have a business that sells tables and chairs. You have brown tables and white tables, and corresponding chairs. Your May sales are 4 brown tables, 6 white tables, 20 brown chairs, and 24 white chairs, which is represented by the matrix $M = \begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$ where the first row is tables, the second row is chairs, the first column is brown items, and the second column is white items. Your June sales are given by the matrix $J = \begin{bmatrix} 6 & 8 \\ 22 & 32 \end{bmatrix}$. What matrix operations would make sense in real life in this scenario? Be prepared to discuss why or why not for each.

10. In the homework due today, did you use something similar to the following critical analysis/reasoning?

Multiply both sides of an equation by the inverse of a matrix

Reorder parenthesis by associativity to pair a matrix with its inverse

Cancel A by its inverse: $A^{-1}A = I_{n\times n}$ or $A^{-1}A = I_{n\times n}$

Identity reduces

11. Evaluate the statement: If A is an invertible $n \times n$ matrix, and $\vec{x}$ and $\vec{b}$ are $n \times 1$ vectors, then the matrix-vector equation $A\vec{x} = \vec{b}$ has a unique solution

12. Evaluate the statement: If $A\vec{x} = \vec{0}$, then is $C(A\vec{x}) = \vec{0}C$?

13. Evaluate the statement: If A is an invertible $n \times n$ matrix, with $n > 1$, and $\vec{x}$ and $\vec{b}$ are $1 \times n$ vectors, then the matrix-vector equation $A\vec{x} = \vec{b}$ has a unique solution

14. Evaluate the statement: If A is *not* invertible and $AB = AC$, must $B = C$?

15. Given $A_{n\times n}$ (square), can $A\vec{x} = \vec{0}$ have only the trivial solution?

16. Given $A_{m\times n}$ (not square), can $A\vec{x} = \vec{0}$ have only the trivial solution?

17. Evaluate statements related to the Hill cipher

18. If the condition number of a square matrix with fractional entries is 3.5×10^6 then...

19. The equation $A\vec{x} = \vec{b}$ has at least one solution for each $\vec{b}$ in $\mathbb{R}^n$ whenever A is an $n \times n$ matrix.

20. If there is a $\vec{b}$ in $\mathbb{R}^n$ such that the equation $A\vec{x} = \vec{b}$ is consistent, where A is $n \times n$, then the solution is unique.

21. If the columns of a 7×7 matrix D are linearly independent, what can be said about the solutions $D\vec{x} = \vec{b}$ for a given 7×1 vector $\vec{b}$ (where $\vec{x}$ is 7×1 too)?

22. If the columns of a 7×6 matrix D are linearly independent, what can be said about the solutions $D\vec{x} = \vec{b}$ for a given 7×1 vector $\vec{b}$ (where $\vec{x}$ is 6×1)