Exam 2 *i-clicker* questions

Here are portions of questions to help you with your notes. The precise wording and ordering may change and we may not have time to cover all of these. The purpose of *i-clickers* is to actively practice concepts, computational strategies, critical & creative thinking, and communication. Making mistakes is integral to the learning process and enriches our understanding as we extend content and clear up misconceptions.

- **Think** about a possible answer(s) on your own.
- **Pair up**: discuss your thoughts in a group. We may reorganize groups at times.
- Prepare to **share** something from your group's discussion. This may take the form of an assertion, question, definition, example, or other connection. It could be something you tried and rejected.
- May be a lag at times—use this to **review** related concepts and examples, and **add** to your notes and the glossary, or get to know your neighbors.

Appalachian's General Education Program prepares students to employ various modes of communication. Successful communicators interact effectively with people of both similar and different experiences and values and in this class you will practice oral and written communication during class by interacting with various peers and me.

Clickers in 1.8, 1.9, and 2.7

1. Define $T(\vec{v}) = A\vec{v}$, where A is $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Then $T(\vec{v})$

2. Define $T(\vec{v}) = A\vec{v}$, where A is $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Then $T(\vec{v})$

3. Define $T(\vec{v}) = A\vec{v}$, where A is $\begin{bmatrix} .5 & .5 \\ .5 & .5 \end{bmatrix}$. Then the range of T, the set of outputs of the linear transformation, is

4. To rotate a figure $\begin{bmatrix} x_1 & ... & x_p \\ y_1 & ... & y_p \\ 1 & ... & 1 \end{bmatrix}$ about a point (-2,3) we can

5. Evaluate statements about linear transformations

6. Let $T : \vec{x} \to A\vec{x}$ be given as a linear transformation arising from a square 2x2 matrix A. Assume that the set of all outputs $\vec{b}$ (from $A\vec{x} = \vec{b}$) is a line. What can we deduce?

7. To turn a car so that it points in the direction of motion, we

8. To keep a car on a curved race track, we can perform the appropriate matrix operations in the following order

9. To rotate Yoda, who was given in row vectors as opposed to column vectors, we made use of

Clickers in 3.1, 3.2, 3.3

1. Which of the following class topics relate to determinants?

2. Which of the following matrices does not have an inverse?

3. Which of the following are true about the matrix $A = \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$

4. By hand, use Laplace expansion as directed $\begin{bmatrix} 5 & 2 & 0 & 0 & -2 \\ 0 & 1 & 4 & 3 & 2 \\ 0 & 0 & 2 & 6 & 3 \\ 0 & 0 & 3 & 4 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$

   Step 1: First expand down the first **column** to take advantage of the 0s. You'll have one nonzero term.

   Step 2: then down the 1st **column** of the resulting 4×4 matrix

   Step 3: then along the 3rd **row** of the 3×3 matrix. The determinant is

5. Evaluate statements about determinants

6. Suppose the determinant of matrix A is zero. How many solutions does the system $A\vec{x} = 0$ have?

7. We find that for a square coefficient matrix A, the homogeneous matrix equation $A\vec{x} = \vec{0}$, has only the trivial solution $\vec{x} = \vec{0}$. This means that

8. Suppose the determinant of matrix A is zero. How many solutions does the system $A\vec{x} = \vec{b}$ have?

9. If A is an invertible matrix, what else must be true?

10. In exercise 3.3 #19, the area of the parallelogram is 8, because that is the determinant of $A = \begin{bmatrix} 5 & 6 \\ 2 & 4 \end{bmatrix}$. Can we find a rectangle that creates a matrix that is row equivalent to A with the same area?

Clickers in 2.8

1. Which of the following statements about are true about the nullspace (or null space) and column space of $M = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$. Note that when M is augmented with a generic vector and reduced to Gaussian, the last row becomes $\begin{bmatrix} 0 & 0 & b_1 - 2b_2 + b_3 \end{bmatrix}$

2. If a matrix is not square, then the column space is a subspace of

3. The definition of a basis is a linearly independent spanning set for V. Which of the following also describes a basis?

Clickers in Chapter 5

1. In Maple we execute
   A := Matrix([[21/40,3/20],[-3/16,39/40]]);
   Eigenvectors(A);
   and obtain the output $\begin{bmatrix} \frac{3}{5} \\ \frac{9}{10} \end{bmatrix}, \begin{bmatrix} 2 & \frac{2}{5} \\ 1 & 1 \end{bmatrix}$
   Write out the eigenvector decomposition. What does the trajectory look like for an initial vector in quadrant 1 that does not begin on either eigenvector?

2. Say that A represents the changes from one year to the next in a system of foxes (x-value) and rabbits (y-value). For most initial conditions, what happens to the system in the longterm?

3. Given a square matrix A, to solve $A\vec{x} = \lambda\vec{x}$ for eigenvalues and eigenvectors...

4. A reflection matrix has eigenvalue(s)

5. An eigenvector $\vec{x}$ allows us to turn matrix

6. A := Matrix([[21/40,3/20],[19/240,39/40]]);
   Eigenvectors(A); $\begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}, \begin{bmatrix} -6 & \frac{6}{19} \\ 1 & 1 \end{bmatrix}$
   What are the relative population sizes in the longterm? [foxes (x-value) and rabbits (y-value)]

7. What does the trajectory look like for an initial vector in quadrant 1 that does not begin on either eigenvector?

8. The eigenvector decomposition for a system of owls (x-value) and wood rats (y-value) is given via $\vec{x}_k = \begin{bmatrix} \text{owls}_k \\ \text{wood rats}_k \end{bmatrix} = a_1(\frac{7}{10})^k \begin{bmatrix} 2 \\ 1 \end{bmatrix} + a_2(\frac{9}{10})^k \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
What happens to the populations in the longterm for most starting positions?

9. What is the trajectory of $\vec{x}_k$ for most starting positions?

10. When do we die off along $y = \frac{1}{2}x$?

11. Could we write out an eigenvector decomposition for a reflection matrix (i.e. are there 2 linearly independent eigenvectors that span $\mathbb{R}^2$)

12. Could we write out an eigenvector decomposition for a projection matrix?

13. Could we write out an eigenvector decomposition for a horizontal shear matrix?

14. If $A_{2\times 2}$ has a nonzero eigenvector $\vec{x}$ satisfying $A\vec{x} = 5\vec{x}$ then A's eigenvectors are

15. If the reduced augmented matrix for the system $(A - \lambda I)\vec{x} = \vec{0}$ is $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ with A as a 2×2 matrix then the non-zero (real) eigenvectors of A are

16. What are the non-zero real eigenvectors of $\begin{bmatrix} \cos\pi & -\sin\pi \\ \sin\pi & \cos\pi \end{bmatrix}$?

17. If we write a **basis** for the eigenspace of $\begin{bmatrix} \cos\pi & -\sin\pi \\ \sin\pi & \cos\pi \end{bmatrix}$, how many vectors does it have?

18. What are the non-zero real eigenvectors of $\begin{bmatrix} \cos\frac{\pi}{5} & -\sin\frac{\pi}{5} \\ \sin\frac{\pi}{5} & \cos\frac{\pi}{5} \end{bmatrix}$?