1. If the columns of a 7×7 matrix D are linearly independent, what can be said about the solutions $D\vec{x} = \vec{b}$ for a given 7×1 vector $\vec{b}$ (where $\vec{x}$ is 7×1 too)?

a) $D\vec{x} = \vec{b}$ always has at least one solution, but we cannot say anything more about the solution or solutions
b) $D\vec{x} = \vec{b}$ always has a unique solution, but we cannot say anything more about it
c) $D\vec{x} = \vec{b}$ always has a unique solution, and I can tell you what it is
d) $D\vec{x} = \vec{b}$ always has infinite solutions
e) $D\vec{x} = \vec{b}$ has no solutions for some $\vec{b}$ and infinite solutions for other $\vec{b}$

2. If the columns of a 7×6 matrix D are linearly independent, what can be said about the solutions $D\vec{x} = \vec{b}$ for a given 7×1 vector $\vec{b}$ (where $\vec{x}$ is 6×1)?

a) $D\vec{x} = \vec{b}$ always has at least one solution, but we cannot say anything more about the solution or solutions
b) $D\vec{x} = \vec{b}$ always has a unique solution
c) $D\vec{x} = \vec{b}$ has no solutions for some $\vec{b}$ and infinite solutions for other $\vec{b}$
d) $D\vec{x} = \vec{b}$ has one solution for some $\vec{b}$ and no solutions for other $\vec{b}$
e) We can reason that $D\vec{x} = \vec{b}$ has 0,1, or infinite solutions as with any linear system, but we cannot specify the solutions any further.

$$AB = \begin{bmatrix} A.\text{col}1B & \ldots & A.\text{col}nB \end{bmatrix}$$

pivot arguments:

- $A_{n\times n}$ must have full pivots to be invertible because it reduces to the identity matrix I so you can make use of the full pivots
- $A_{n\times n}$ must be missing both row and column pivots if it isn't invertible so you can make use of missing pivots
- A not square must be missing a row pivot or a column pivot (but not necessarily both)

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Multiply both sides: $A^{-1}(A\vec{x}) = A^{-1}\vec{b}$
Reorder by associativity: $(A^{-1}A)\vec{x} = A^{-1}\vec{b}$
Cancel A by its inverse: $I_{n\times n}\vec{x} = A^{-1}\vec{b}$
Reduce identity: $\vec{x} = A^{-1}\vec{b}$

- Hill Cipher: message goes in as column vectors
- A.[uncoded message] $=$ [coded message]

$$A \begin{bmatrix} \text{col1=start of message} & \dots & \text{coln=end of message} \end{bmatrix}$$

- [uncoded message] $= A^{-1}$.[coded message]
- Vulnerable because of its linearity—intercept enough vector correspondances (uncoded and coded)

- Condition Number Roundoff Errors: asymptotically worst case loss of accuracy: 10^{19}: 19 digits=k, order
- Using r digits gets at least $r - k$ accuracy [ex: 21-19=2]
- hard to numerically distinguish between a non-invertible matrix and one with a large condition number

Catalog description: A study of vectors, matrices and linear transformations, principally in two and three dimensions, including treatments of systems of linear equations, determinants, and eigenvalues. Prerequisite: MAT 1120 or permission of the instructor.

Course Goals

- Develop algebraic skills
- Develop mathematical reasoning and problem solving
- Develop spatial visualization skills
- Learn about some applications of linear algebra
- An introduction to a computer algebra software system as it applies to linear algebra

Mapping of the Topics in the Catalog Description to the Text

Systems of Linear Equations: 1.1, 1.2, 1.5
Vectors: 1.3, 1.4, 1.7
Matrices: earlier +2.1, 2.2, 2.3
Linear transformations 1.8, 1.9 (1-1 & onto eliminated), 2.7, 6.1
Determinants: 3.1, 3.2, 3.3
Eigenvalues 2.8, 5.1, 5.2, 5.6