

theoretical multiplication arguments:

$$AB = \begin{bmatrix} A.\text{col}1B & \dots & A.\text{col}nB \end{bmatrix}$$

OR

Multiply both sides on side it makes sense: $A^{-1}(A\vec{x}) = A^{-1}\vec{b}$

Reorder parentheses by associativity: $(A^{-1}A)\vec{x} = A^{-1}\vec{b}$

Cancel A by its inverse: $I_{n \times n}\vec{x} = A^{-1}\vec{b}$

Reduce identity: $\vec{x} = A^{-1}\vec{b}$

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- $A_{n \times n}$ must have full pivots to be invertible because by above $A\vec{x} = \vec{b}$ never inconsistent (full row pivots) and A is square so full column pivots too, and A reduces to $I_{n \times n}$

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- $A_{n \times n}$ isn't invertible

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- $A_{n \times n}$ isn't invertible must be missing row and column pivots
- A not square must be missing a row pivot or a column pivot (but not necessarily both)

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Catalog description: A study of vectors, matrices and linear transformations, principally in two and three dimensions, including treatments of systems of linear equations, determinants, and eigenvalues. Prerequisite: MAT 1120 or permission of the instructor.

Course Goals

- Develop algebraic skills
- Develop mathematical reasoning and problem solving
- Develop spatial visualization skills
- Learn about some applications of linear algebra
- An introduction to a computer algebra software system as it applies to linear algebra

Mapping of the Topics in the Catalog Description to the Text

Systems of Linear Equations: 1.1, 1.2, 1.5

Vectors: 1.3, 1.4, 1.7

Matrices: earlier +2.1, 2.2, 2.3

Linear transformations 1.8, 1.9 (1-1 & onto eliminated), 2.7, 6.1

Determinants: 3.1, 3.2, 3.3

Eigenvalues 2.8, 5.1, 5.2, 5.6

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- The order k in scientific notation (10^k) is useful, rather than the number, which can be different in different programs
- measures asymptotically worst case scenario that we may lose up to k digits in roundoff errors
- issue with decimals in the matrix, not with Maple
- using r digits gets at least $r - k$ accuracy [ex: $21-19=2$]



http://c.asstatic.com/images/1639965_634936497651212500-1.jpg

- Hill Cipher: message goes in as column vectors
- $A \cdot [\text{uncoded message}] = [\text{coded message}]$

$$A \left[\begin{array}{c} \text{col1=start of message} \quad \dots \quad \text{coln=end of message} \end{array} \right]$$

- $[\text{uncoded message}] =$

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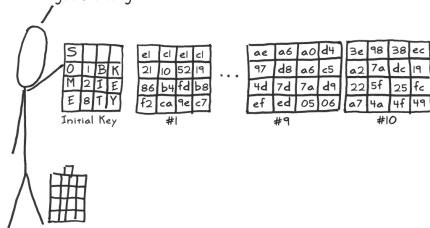
$$A \left[\begin{array}{l} \text{col1=start of message} \quad \dots \quad \text{coln=end of message} \end{array} \right]$$

- $[\text{uncoded message}] = A^{-1} \cdot [\text{coded message}]$

- Vulnerable because of its linearity—intercept enough vector correspondances (uncoded and coded)

Key Expansion: Part 1

I need lots of keys for use in later rounds. I derive all of them from the initial key using a simple mixing technique that's really fast. Despite its critics,* it's good enough.



* By far, most complaints against AES's design focus on this simplicity.

<http://www.moserware.com/2009/09/stick-figure-guide-to-advanced.html>

A Stick Figure Guide to the Advanced Encryption Standard (AES)