MAT 2240 Video Interactions

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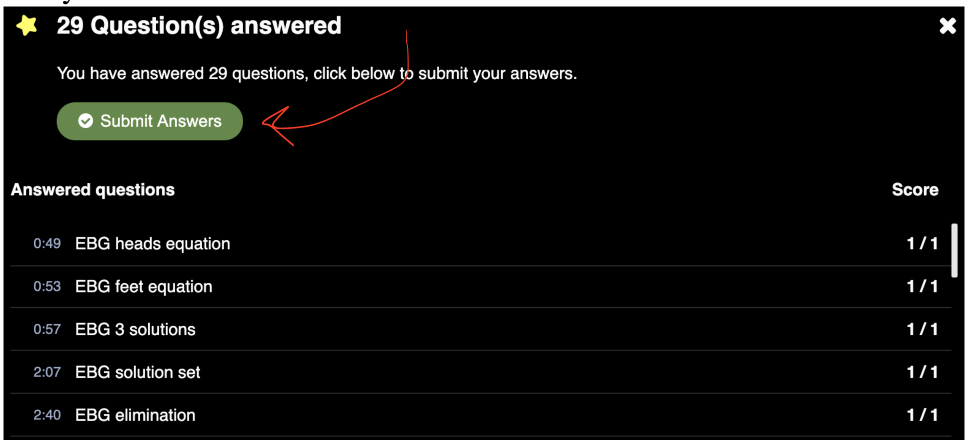
These are interactions in the interactive videos we created*. In the interactive video, the video pauses, asks a question, and requires a response to proceed. To earn credit we watch the entire video and submit the correct answers via the green "Submit Answers" button at the very end of the video, the one that shows all the questions we have answered—we use the check feature on interactive questions in order to help and can redo the responses until they are correct.



Each video includes directions to "pause regularly to take notes especially on definitions, concepts, examples, computations, theorems, visualizations, Maple work and any remaining questions." Above each video we include resources needed such as Maple files.

Contents

1.1 interactive video

- What is the equation for the heads?
- What is the equation for the feet?
- Solve the system $x + y = 17, 4x + 2y = 48$ write the solution in your notes.
- What is the solution set for the Evelyn Boyd Granville challenge problem $x + y = 17, 4x + 2y = 48$?
- What operation uses x in $x + y = 17$ to eliminate the term below it in $4x + 2y = 48$?
- Consider how Gaussian elimination works here and write it in your notes.
- Consider the matrix and Gaussian elimination notation.
- Open the Maple file 11intro.mw, which is available just above this video.
- Execute the packages and implicitplot lines in Maple and make sense of what you see.
- If r_1 is $[1\ h\ 0]$ and r_2 is $[h\ 1\ 0]$ then what is $r_2' = -hr_1 + r_2$?
- Consider the Gaussian elimination reduction as well as the number of solutions. How many solutions does this system have?
- Consider how all this shows infinite solutions when h is 1 or -1 and one unique solution otherwise and write the reasoning in your notes.
- Open `https://www.desmos.com/calculator/h6ocyou5f0`. Engage with the Desmos slider yourself. What are the solutions of $x + hy = 0$ and $hx + y = 0$ when $h = -1$?
- What is wrong with the ReducedRowEchelonForm of $r_1 = [1\ 0\ 0]$ and $r2 = [0\ 1\ 0]$ for this system?
- What are the possible solutions of a linear system with two equations and two unknowns? Can you provide examples? More generally, why do these represent all the possibilities?
- If possible, draw a picture of a linear system with two equations and three unknowns that has a unique solution. If this is not possible, explain why not.
- Can 2 planes intersect in a unique point?
- For each visualization of 3 equations and 3 unknowns, how many solutions does each have? Where in the room do we see them?

- Sketch the images in your notes and annotate where we can find them in real-life.
- Consider the augmented matrix corresponding to the system of 3 equations and 3 unknowns $3x - z = 3, 2x - y + z = 8$, and $x + 2y + 3z = 9$ and write it in your notes.
- Write the first step of elimination in your notes. What elementary row operation comes next?
- Consider the elimination steps and also what does the last row reduce to?
- Consider the back substitution steps and solve for x by substituting $z = 3$ from row 3 and $y = -1$ from row 2 into equation 1: $x + 2y + 3z = 9$. What do we obtain?
- What does replacement do to a plane geometrically?
- Consider the Maple syntax.
- Compare and contrast implicitplot3d, GaussianElimination, and ReducedRowEchelonForm (rref)
- How many solutions does this system have?
- How many solutions does the modified system have?

1.2 interactive video

- What type of row operation would you use to zero out the entries below the highlighted 1?
- Which is a valid replacement operation in strict Gaussian?
- A pivot position in the matrix corresponds to a row–leading entry in row echelon form. Is the 2 in a pivot position?
- In your notes, write the original system, its augmented matrix, the reduction, circle the pivots 1 and -4 in your notes and identify the pivot columns too. If you have any questions, write those down too.
- Write how we solved the system in your notes and consider the geometry.
- Open 12intro.mw in Maple, found above the video.
- Execute the commands, including the packages. Then modify the -2 .. -1.8 in the 2nd copy in 12intro.mw to see the parameter in action, how it moves.
- Open up https://www.geogebra.org/3d/ybgtp3vg above the video and use the slider to see the free parameter in action.
- If a row of all zeros was above, how could we get it below?
- Consider that we could use interchange if we needed to move rows around so that the leading entries move to the right as we move down the matrix
- Consider how row echelon form requires zeros below the diagonal and how it can be obtained from the strict method of Gaussian using only replacement and, only if needed, interchange. Write down the definition.
- Which is not in ref?

- For a), b), and c), consider how many solutions, if any, and how many free variables, if any
- Consider the placement of the pivots (row–leading entries in row echelon form) for inconsistent and unique solutions in a) and c).
- How many pivots and pivot columns does each system of 3 equations and 3 unknowns have?
- Consider why any linear system has 0, 1 or infinite solutions
- In your notes, summarize the pivot argument to explain why any linear system has 0, 1 or infinite solutions
- Can an underdetermined system have a unique solution?
- We'll focus on REF for by-hand work. As Maple moves from REF to RREF, what type of row operation would Maple use to make sure that the leading entry in a given row is 1?
- Consider how the 1 has been used to eliminate what comes above it.
- How many solutions do we have here?
- At most, how many flops will be needed to take a 3×4 matrix from ref to rref?
- Consider how many solutions, if any
- In your notes, set $x_3 = s$ and $x_4 = t$ and then use back substitution to solve for x_2 and then x_1 to finish parameterizing the solutions. Show work.
- Consider how we first checked for consistency, then identified free variables as those missing pivots in a consistent system, then worked from the bottom pivots back upward from row 2 to row 1 to solve for them.
- Which is true about Gaussian on the augmented matrix?
- Write out the row echelon form and then consider b and c.

Maple intro

- What should we do about Maple's forgetfulness and lack of forgetfulness?
- Open Mapleintro.mw (the file is above the video) and execute the 5 commands. If you downloaded Maple to your computer, or are working on a campus computer with Maple on it, then you can open it from there. Otherwise, open it from UDesk.
- What's happening here? Why is the program spitting back the commands without executing them?
- After executing the packages, do we need to execute or re-execute the other command lines?
- Consider how all 3 methods show 1 unique solution in different ways.
- In your Maple document, choose a different first equation (different than $x + y = 17$ or $x + 2y = 17$). Modify the first equation in the implicitplot command and the x values so that you see the intersection point. Be sure to use * for multiplication.
- In your Maple document, modify the augmented matrix to match your first equation in your implicitplot command and execute it to see the new augmented matrix. Then re-execute the Gaussian and rref commands below it.

- Consider how you will collate your Maple work and your by-hand work into one fullsize multipage PDF for the next assignment coming up and for problem sets.
- Why bother with Maple?

1.3 interactive video

- Sketch our standard mathematical axes in $\mathbb{R}^2$ and plot the vector $\begin{bmatrix}1\\2\end{bmatrix}$ as shown and then also $\begin{bmatrix}-1\\1\end{bmatrix}$. Which quadrant is the vector $\begin{bmatrix}-1\\1\end{bmatrix}$ in?
- Which coordinate sticks out of the board in $\mathbb{R}^3$ in linear algebra?
- In your notes, sketch our standard mathematical axes in $\mathbb{R}^3$ and plot the vector $\begin{bmatrix}1\\-2\\.5\end{bmatrix}$ as shown and then also $\begin{bmatrix}1\\2\\3\end{bmatrix}$.
- Compare your sketch of the vector $\begin{bmatrix}1\\2\\3\end{bmatrix}$ and make any corrections.
- In your notes, plot the original and scaled vector and label them.
- In your notes, write the slope for vectors $\begin{bmatrix}x\\y\end{bmatrix}$ and consider why the slope of the line the vector $\begin{bmatrix}3\\1\end{bmatrix}$ is on is $\frac{1}{3}$, i.e. the line is $y = \frac{1}{3}x$.
- Open https://www.geogebra.org/m/bztwgftf and drag the slider to solidify the scalar multiplication of $\lambda \begin{bmatrix}1\\-2\\.5\end{bmatrix}$.
- How does $\lambda = -2$ scale the vector?
- Start by plotting the vectors $\begin{bmatrix}1\\2\end{bmatrix}$ and $\begin{bmatrix}2\\1\end{bmatrix}$ add component wise and then plot the addition. Consider what the addition is geometrically/physically.
- Open https://www.geogebra.org/m/dr8mjdeq and drag the sliders to solidify the addition of a variety of different vectors in 3-space
- Adding two vectors on different lines gives what?
- Write the definition of linear combination in your notes and consider how the scalar multiplications and additions are both a part of it
- Consider the visualization of the parallelepiped and linear combinations of the vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$,
- What represents the system?

- Write the two forms of the system in your notes
- Open 13intro.mw
- Execute the commands I did. What does the reduction show us? How many solutions do we have?
- Write the custom mix, the equals column starting with 36, as a linear combination of the first two fractional vectors using the weights of 40 and 60.
- How many solutions do we have?
- Should we use decimals or fractions in Maple?
- Consider the equivalent linear representations and how the definition of linear combination connects.
- How does a vector equation turn into an augmented matrix?
- Write the definition of span
- Sketch some of the linear combinations and convince yourself that the entire set of linear combinations, the span, is all of $\mathbb{R}^2$.
- What is the span of these two vectors that are on the same line
- What will the span of $\vec{v}$ and $\vec{w}$ be in this case?
- Write the vector equation and corresponding augmented matrix for the span of $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \right\}$
- Consider why generic vectors $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ in the span must satisfy the equation $b_1 - 2b_2 + b_3 = 0$.
- In your notes, apply the 3rd step in Gaussian and simplify.
- $1 - 2(2) + 3 = 0$ satisfies the consistency equation for the span $b_1 - 2b_2 + b_3 = 0$ so $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is in the span. Check the coordinates by plugging x for b_1, y for b_2 and z for b_3. Check the equation to determine whether $\begin{bmatrix} 3 \\ 3 \\ 5 \end{bmatrix}$ in the span?
- What does this spacecurve command plot?
- In `https://www.geogebra.org/m/gw7gtznx` engage with the sliders yourself to visualize lots of linear combinations. Then try and turn it head on to see the span is a plane in 3-space.
- Execute the commands in your Maple file and turn the $b_1 - 2b_2 + b_3 = 0$ plane head on and see the vector $\begin{bmatrix} 7 \\ 8 \\ 100 \end{bmatrix}$ sticking outside of it, showing it is not in the span of the other vectors.
- Consider what club they can now form?
- Which is true about the collection of vectors in $\mathbb{R}^2$?

- What does the span form?
- Which is true about the collection of vectors in $\mathbb{R}^3$?
- Which is true here?
- Execute the commands in Maple and consider what happens when we correctly use Gaussian.
- Compare the spans.

1.4 interactive video

- Write the example of $A\vec{x}$ in your notes to get used to the matrix-vector multiplication
- In your notes, write 3 representations of the system: augmented matrix, vector equation, matrix-vector product
- Compare these with your notes to make any corrections, if any are needed.
- Which of the matrix-vector products is not possible?
- Compute and write the matrix-vector product in your notes.
- Write the definition of I_n in your notes.
- Why did we convert to fractions in the house and deluxe blends?
- Given that there are vectors in $\mathbb{R}^4$ that are not in the span of the vectors h and d, what is the answer to all four of the questions listed?
- Write the ref form in your notes and the equation for consistency. Next, what property of the ref form of the augmented matrix allowed for some of the coffee mixes to be impossible to make?
- Check if $\begin{bmatrix} 2 \\ 1 \\ 2 \\ 3 \end{bmatrix}$ satisfies the span condition equation $-2r + 2c + k + 0s = 0$
- Write the Connections Theorem in your notes
- Consider a), b), and c) for A
- Consider the statements for D
- Open GeoGebra 1 `https://www.geogebra.org/m/qdqfadc2` and drag the c_1 and c_2 sliders to see many linear combinations in the span and then try to turn them head on to see they all lie in the same plane.
- Write the four different representations in your notes
- In your notes, write the steps of Gaussian elimination to reduce to row echelon form. Is $\vec{w}$ in the plane spanned by $\vec{u}$ and $\vec{v}$?
- Open GeoGebra 2 `https://www.geogebra.org/m/u2mdhker` above the video and turn the plane head on to see w sticks out of it a teeny bit

- Write the definition of dot product in your notes. Next, what is the dot product of the vectors $\begin{bmatrix}1\\0\\0\end{bmatrix}$ and $\begin{bmatrix}1\\2\\3\end{bmatrix}$
- Write in your notes, compute the dot products of the rows of A with $\vec{x}$, and then consider how the linear combinations of the columns of A with the weights from $\vec{x}$ come in.
- Compute $A\vec{x}$ in 2 ways with $A = \begin{bmatrix}1 & 2\end{bmatrix}$ and $\vec{x} = \begin{bmatrix}3\\4\end{bmatrix}$. Write the 2 methods in your notes as you compute $A\vec{x}$ in 2 ways. Use linear combinations as well as dot products.
- Compare with your notes and make any corrections there.
- Which is true about Fortran and C?
- Consider how addition and scalar multiplication come in here
- Consider the multiple perspectives here.

1.5 interactive video

- In your notes, apply strict Gaussian to reduce and then write the solutions and write in your notes
- In your notes, write the vector parametrization. Then, what's the geometry of the solutions?
- Open 15intro.mw above, execute the commands in Maple, scroll back up, and drag the plot as directed there to see and internalize the 2, 1, and 0 coordinates.
- Open up GeoGebra #1 $t\vec{v}_1$ above the video and slide the parameter t

  https://www.geogebra.org/m/yscdejye
- In your notes, write the linear algebra definitions of trivial and nontrivial
- Is t times a vector always a line through the origin?
- Open up GeoGebra #2 $s\vec{v}_1 + t\vec{v}_2$ and slide both sliders. Then see if you can turn to a head on view where they are all shown as in the same plane. What is the geometry of $sv_1 + tv_2$ when the 2 vectors are not multiples?

  https://www.geogebra.org/m/bbsqnr4e
- Write out the solutions, if any?
- In your notes, write out the solutions as a vector and then the separation into 2 vectors
- What is the geometry of $tv_1 + v_2$?
- In your notes, sketch the picture of $t\vec{v}_1 + \vec{v}_2$ with the various t values, arrowheads, and labels too.
- separate out and factor the vector $\begin{bmatrix}1+2s+3t\\s\\t\end{bmatrix}$
- Use strict Gaussian elimination, leaving row 1 alone, to put the matrix in row echelon form

- Consider different h values, write solutions in parameterized vector form in your notes, and consider the geometry
- Write the equations of the lines in your notes, $y = -x$ and $y = x$, next to the parameterized vector forms.
- By-hand and in your notes, solve and write the solutions in parametric vector form as you select which one:
- If you haven't already done so, show by-hand work in your notes to show why b) is the correct response.
- Use the 15intro.mw file to turn the planes so that you can internalize the visualizations of the vectors and planes. Then sketch a couple of views in your notes.
- How many solutions does a homogeneous linear system have?
- What is common to all homogeneous systems ($A\vec{x} = \vec{0}$ systems), whether they have 1 or infinite solutions?
- What is the geometry of $c_1\vec{v}_1 + \vec{v}_2$?
- Write in your notes and consider how we are switching from rows to columns to rows to columns! Then open up GeoGebra #3 $s\vec{u} + \vec{v}$ above the video, a nonhomogeneous version of this.

  https://www.geogebra.org/m/nftf4put

1.7 interactive video

- visually inspect that $\vec{w} = 2\vec{u} + \vec{v}$ and that $\vec{w}$ is redundant
- Which of the following form a set of independent directions for the line $y = x$?
- Which of the following form a set of independent directions for $\mathbb{R}^2$?
- In the image on the left of the parallelepiped, which form a set of independent directions for $\mathbb{R}^3$?
- Open https://www.geogebra.org/m/gw7gtznx from above the video, slide the sliders and then consider the span of the vectors as well as whether the 3 vectors are independent.
- Assume $\vec{w} = \vec{u} - 2\vec{v}$. Which of the following are also true?
- Write the definition of linear independent in your notes.
- In your notes, set up the definition of linear independence as it applies to these 3 vectors.
- Do we have only the trivial solution?
- Write out the solutions in parametric vector form like we usually do.
- Since we have all nonzero weights in our linear dependance relations, consider how to write any one of these three vectors in terms of the other two by moving it to the other side and dividing by the nonzero weight. This won't always be possible because we can have redundancy with some but not all nonzero weights, although it is possible here.
- Consider what it means geometrically that these 3 vectors are not linearly independent
- Open 17intro.mw from above the video and execute the commands

- Can we turn them head on in the same plane?
- Consider the linear systems we would use to determine linear independence here
- With h there, what can we use to reduce without introducing errors?
- Which h have the column vectors of the coefficient matrix linearly independent.
- Consider how we can tell by visual inspection that that columns are not linearly independent
- Write this pivot argument for linear independence in your notes
- If we have 3 vectors in $\mathbb{R}^2$ can they be linearly independent?
- Why do columns 1, 3, and 5 form a linearly independent set?
- Consider how these relate to span.
- For linear independence, the augmented matrix reduces to a matrix with pivot position in every...
- Consider why linear independence requires full column pivots in the coefficient matrix while span for the entire space requires full row pivots in the coefficient matrix
- Consider the algebra and geometry of the column vectors of A for linear independence and span.

2.1 interactive video

- What size is this matrix?
- Think about digital images. Consider when are 2 matrices the same?
- What is $5A$?
- What is $A + B$?
- In your notes, write the A and B matrices and what they stand for (a_{1i} the profit of one unit of product i, b_{ij}=the number of units of product i shipped to outlet j)
- Does BA make sense here?
- If we have a 2×3 matrix B then how many entries would a column vector $\vec{x}$ need for $B\vec{x}$ to be defined?
- Consider profit here. Does $3.75 \times 100 + 3.75 \times 125$ make sense?
- Would $3.75 \times 100 + 7 \times 125$ make sense here?
- Multiply and write in your notes.
- What is A times the 2nd column of B?
- Will the multiplication of a 2×2 matrix and a 4×2 matrix be defined?
- What must be the same for AB multiplication?
- What is the size of AB when A is 2×3 and B is 3×2?

- which method would be faster if we are only interested in the a_{32} entry of a matrix?
- Multiply the matrices and write in your notes.
- What is the 21 entry of AB?
- Write out the multiplications using both methods.
- Next, answer the question
- Which make sense in real life?
- Consider the real-life meanings of a), b), and c)
- Which is an algebraic property of matrix multiplication?
- Evaluate the statement
- If the sizes match up, what is $C\vec{0}$?
- What happens to row 2 of A in A^T, the transpose of A?
- What is the transpose of the transpose of A?

2.2 interactive video

- What are the steps to solve $3x = 5$?
- Consider how associativity plays a role
- Write out the steps and the reasons for the steps
- Open 22intro.mw above the video and execute the commands
- Think about the methods we could use to solve the system
- Multiply that out
- Consider the implications of saving time with the inverse method when we keep the same coefficient matrix day to day and only change $\vec{b}$
- Multiply the two matrices by hand and write that in your notes. What is the last row?
- Would it be the same if we reversed the order so that the generic matrix was on the left rather than the right?
- Multiply the 2nd matrix product by hand and write what elementary row operation it is
- Multiply the 3rd matrix product by hand and write what elementary row operation it is
- Write the matrix inverse that will undo A?
- Consider how the inverse is the row operation applied to the identity matrix
- What is the inverse of B?
- fill in: every number but _____ is invertible

- Consider how $[AI]$ turns to $[IA^{-}1]$ via Gauss-Jordan reduced row echelon form when the inverse exists
- Which is true about any matrix with a row of 0s?
- Consider the socks-shoes analogy for the inverse of a product here
- Consider why the columns of an invertible matrix are linearly independent
- why is that true?
- Write out that matrix algebra argument to apply the inverse to $A\vec{x} = 0$

2.3 interactive video

- What can we say about these for an invertible matrix?
- Write the argument and its reasons that is shown here in your notes
- If A is invertible can $A\vec{x} = \vec{b}$ be inconsistent?
- Generally, what does a linear system never being inconsistent tell us about pivots, where pivot is as usual the first nonzero entry in a row after elimination?
- If A is square where we have the same number of rows and columns (but not necessarily invertible), what does that tell us about pivots?
- Write down a matrix that is not square but is in row echelon form. Consider whether it is missing a row pivot, a column pivot, or both
- Write these statements in your notes
- Write these 6 matrices and their letters in your notes
- C is a square matrix so consider the statements of the invertible matrix theorem for C (my favorite example).
- For the matrix D, consider the 5 statements from n pivot positions to span $\mathbb{R}^n$
- For the matrix E, consider the 5 statements from n pivot positions to span $\mathbb{R}^n$
- Consider how $E\vec{x} = \vec{b}$ has inconsistencies for some $\vec{b}$
- What do the columns of matrix F span?
- When are the statements logically equivalent?
- If A is square, can $A\vec{x} = \vec{0}$ have only the trivial solution, and which would hold in this case?
- Write down the argument
- If A was not square, can $A\vec{x} = \vec{0}$ have only the trivial solution, and which would hold in this case?
- Does the system have a unique solution?
- Open up 23intro.mw above the video

- What does the condition number tell us about the identity matrix?
- Consider why the modification of my favorite example gives an invertible matrix
- Which is true about condition number?

2.8 interactive video

- What is $t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ where t ranges over all reals?
- Consider how we could take any 2 vectors on the $y = x$ line so that their sum is still on the $y = x$ line.
- Take a vector on the line $y = x$ and a real number. Multiply them. Is the result still on the same line?
- Add $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ and check if the result is still on the same line $y = x + 1$. What is the equation of the line the sum is on?
- Consider what are the subspaces of $\mathbb{R}^3$
- Why is the set containing my 3 favorite vectors NOT a basis?
- Consider the definition of the span of a set of vectors and how that means that the span will satisfy the subspace conditions that addition and scalar multiplication stay inside the space
- Consider why any plane will have dimension 2.
- How do we go from a basis to the entire subspace it generates?
- Consider what a basis for the $xy0$-plane in 3-space could be.
- Will $\{$Orange, Red, Blue$\}$ be a basis for club $xy0$-plane in 3-space?
- Write down these definitions and consider how we have solved for these in the past.
- Circle the pivots and identify the pivot columns of the original matrix, not the reduced one.
- In your notes, parameterize and write out the solutions to $A\vec{x} = \vec{0}$, just like we did back in Chapter 1.
- In your notes, fill in anything you don't already have from this example any write down any questions.
- Consider how we are moving from rows to columns to rows to columns and that the spaces sit inside different Euclidean spaces.
- Write down A and your responses to a), b), and c)
- Compare your response with mine for the column space.
- Write your responses to d), e) and f). Don't forget to augment with the $\vec{0}$ vector first.
- Compare your response with mine for the nullspace.
- Consider these applications of null space and column space.

- Which are true?
- Open 28intro.mw above the video and execute the commands
- Now that we have gone through this using Maple, consider why d) is true.
- Plug each vector's coordinates, one at a time, into $b_1 - 2b_2 + b_3$ to see if it equals 0.
- Consider Maple's work and idiosyncrasies related to the basis it gives versus the entire space and as compared to our by hand work.
- Which is true?
- If A is 2×3 then the null space is a subspace of what?

2.9 interactive video

- Consider the Gaussian steps and then respond. Do these vectors span 3-space?
- Why do we augment with $\vec{0}$ to show linear independence?
- Are the column vectors of my favorite matrix with the digits 1 through 9 a basis for 3-space?
- Does every basis for a space have the same number of vectors?
- Write the definitions of the rank and the nullity.
- Consider the fractional replacements and why the rows reduce the way they do.
- What is the rank of the matrix?
- What is free here?
- Consider the back substitution so far and continue with the back substitution. What is x_1?
- Consider the algebra and geometry of the column space, rank, null space and nullity.
- Write the rank nullity theorem and consider why it holds.
- What is the geometry of the column space?
- Apply the rank-nullity theorem to the 3×3 projection matrix
- Consider how many pivots the nxn identity matrix has and how many free variables $I\vec{x} = \vec{0}$ has. Which is the rank-nullity theorem where rank+nullity=number of columns
- Consider why $\vec{0}$ is not linearly independent: How many solutions does $c\vec{0} = \vec{0}$ have?
- Use by-hand Gaussian and then apply the rank-nullity theorem.
- What is the geometry of the column space?
- Why was $-s + 12t$ substituted in to find $x_1 = 2(-s + 12t) - 4t$?
- Open up 29intro.mw above the video

- Is there an error here?
- Consider how the pivot columns satisfy the spanning equation and how Maple's vectors satisfy it too but how vectors in the reduced matrix do not.

1.8 and 1.9 interactive video

- To see that the range of the linear transformation is the span of the columns of the matrix A, which definition of multiplication is useful?
- When we solved $A\vec{x} = \vec{b}$ for $\vec{x}$, what were we doing in the language of linear transformations?
- What is $T(x)$?
- Fill in the blanks
- Compute the output or image of the vertical shear linear transformation for each of the three inputs by performing matrix multiplication like we did in 1.4 and 2.1
- Create the two plots and reflect on the vertical shear transformation
- Multiply the shear matrix times a generic vector
- If $k < 0$ in a vertical shear, is the wind blowing the square up or down?
- Apply the projection transformation to the three inputs
- What does the matrix of all entries 1/2 do to coordinates?
- Create the two plots and consider the projection onto $y = x$ transformation
- In your inputs diagram, drop a perpendicular to the line of projection $y = x$, from $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, i.e. perpendicular to $y = x$ and through $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Notice the base of the right triangle you will create is the output vector $T(\begin{bmatrix} 0 \\ 1 \end{bmatrix})$
- What does a matrix that is NOT invertible do to the plane?
- Is $\begin{bmatrix} 5 \\ 5 \end{bmatrix}$ in the range of this transformation?
- What is the multiplication of A acting on the unit x-axis?
- Write the vector $\vec{x}$ as a linear combination of the vectors $\vec{u}$ and $\vec{v}$ like we did back in 1.3
- Write the vector x as a linear combination of the vectors u and v like we did back in 1.3
- What would $T(u + v)$, be?
- Write the images of the unit axes and then put them together to form the matrix of the linear transformation
- To see why, this right triangle is a $45°, 45°, 90°$ isosceles right triangle with hypotenuse 1, so apply the Pythagorean theorem to solve for the other sides x via $x^2 + x^2 = 1$

6.1 interactive video

- Compute the norm of $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$
- Consider why the matrix multiplication of 2 vectors is not defined.
- What happens if we multiply $\vec{u}^T\vec{v}$ using regular matrix multiplication here?
- Using the picture on the left, what is $\cos(\theta)$?
- Write down the algebraic definition of dot product, the definition of norm, and the geometric definition of dot product
- In the picture on the right, what vector has something to do with the dotted line?
- When is the dot product of two nonzero vectors equal to 0? When the angle between them is...
- Are $\vec{u}$ and $\vec{v}$ orthogonal?
- compute the lengths of these vectors
- Calculate a unit vector, a vector of length 1, in the $\vec{v}$ direction and compare it to the length of $\vec{u}$. Which vector is longer?
- Download 61intro.mw and open it in Maple.
- What is the angle between the column vectors in the rotation matrix?
- Compute the length or norm of $\vec{v}$
- Are the columns of a matrix representing reflection across a line in 2-space orthogonal?
- What is $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ applied to a generic vector $\begin{bmatrix} x \\ y \end{bmatrix}$ under matrix multiplication?
- Either by-hand or in Maple, compute the dot product of the vector $\begin{bmatrix} -\frac{11}{7} \\ -\frac{13}{7} \\ 1 \end{bmatrix}$ with the 2nd column vector $\begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}$.
- Modify plot 1 to match with the coefficients of the spanning plane we found, hit return and then turn the plane head on to see the orthogonality.
- Consider how this generalized inner product for spacetime has a 4×4 symmetric matrix g_{ij} inserted inside the square root between $\vec{v}^T$ and $\vec{v}$

2.7 interactive video

- Consider why the reflection about the x-axis, which fixes x and sends y to $-y$, preserves the x figure centered at the origin and shown just above my head.
- In your notes, sketch a figure that isn't symmetric. Then let each A from spikedmath act on your figure via left multiplication of Afigure. Sketch the results.
- What is the multiplication?
- What are the names of the linear transformations and what are the matrix representations of them?
- What are the names of the linear transformations and what are the matrix representations of them?
- What are the matrix representations?
- What important transformation is missing?
- Can you find a 2×2 matrix that will translate the origin?
- What are the 6 question marks that act on $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ to give translation?
- What will the bottom row [0 0 1] do to the last coordinate?
- Which are true about linear transformations?
- What's the problem?
- To rotate about a point $(-2, 3)$ we can
- What would let us move along the road?
- What from calculus would give us the direction?
- Consider how norm and orthogonality come in
- Which composition will keep a car on the road pointed in the correct direction?
- to turn a car so that it points in the direction of motion we
- to keep the car on the $(\cos(t), \sin(2t))$ track
- What does the matrix on the right do?
- Consider how the spaceship will move?
- What could these represent?
- Do you see the problem with either of the matrices?
- How could we change the rows to columns?
- Calculate the transpose of the rotation.
- to rotate Yoda, we
- Digitize your preferred name

- If CodedMessage=CodingMatrix.Message, how would someone decode it?
- Consider this application of inverse, associativity, and the identity
- For the Hill cipher

3.1, 3.2, 3.3 interactive video

- Where could determinants be useful?
- Which matrix has orthogonal column vectors, unit length vectors, and determinant 1?
- Open up 313233intro.mw above the video and execute the commands
- Execute the 4×4 commands in Maple. Where does the determinant appear in the matrix inverse of the 4×4 in Maple?
- What is the pattern between n, the size of the $n \times n$ matrix, and the number of terms in the determinant?
- Can we use a diagonal method on a 4×4 matrix for the 24 terms in the determinant from Maple?
- We are going to use the the sum of the entries a_{2j} times cofactors to compute the determinant. Let's let $i = 2$ and $j = 1$ in my favorite example here. What is the entry a_{21} ?
- The difference between the cofactor and the minor is only the $(-1)^{i+j}$. They are both numbers obtained from determinants of smaller matrices in a recursive method. For example, the determinant attached to a_{21} is obtained by a matrix where we have removing row 2 and column 1 of the matrix. What is the new first row?
- Consider how the expansion along the second row works and compute it
- Write out the expansion along column 1
- Compute the determinant using the expansion along column 1. How does it compare to the other expansion?
- Where did the other 3 cofactors from the first row go?
- Does the $(-1)^{i+j}$ stay the same when we expand along a row or column?
- If we wanted to calculate the determinant of the 1st 4×4 matrix where would the top left -2 be located?
- Write out the cofactor/Laplace expansion and use it to compute the determinant of the 5×5?
- Compare the determinant before and after the replacement using $ad - bc$
- Compare the determinant before and after swapping row 1 and 2
- Compare the determinant before and after scaling row 2 by a nonzero constant
- Compare the determinant of the original with the transpose
- What is the determinant of cA, where A is $n \times n$?

- Compute the determinant of one of the triangular matrices by hand (notice they are transposes so expanding along the column of one is the same as expanding along the row of the other, which is why their determinants will be equal.) Pick a row or column with lots of 0s to expand along in one of them, and then continue the expansion until you have computed the determinant.
- What is the determinant of a triangular matrix?
- How does the determinant of the inverse of the 2×2 matrix compare with the original?
- Why is that?
- What is the determinant of the $n \times n$ identity matrix, I_n?
- Write the argument that the determinant of an invertible matrix can't be 0.
- What is an invertible matrix row equivalent to?
- If $|A| = 0$ how many solutions does $A\vec{x} = \vec{0}$ have?
- If $A\vec{x} = \vec{0}$ has only the trivial solution for A square, which is true?
- Apply the first step of strict Gaussian and sketch the resulting row vectors
- Apply what we did in chapter 1 here
- Sketch the shear in your notes
- What is the absolute value of $|A|$ for a 2×2 invertible matrix?
- What is the geometric meaning of determinant for a 3×3 invertible matrix?
- What happens when we apply replacements to a 3×3 invertible matrix to reduce close to rref but do not scale?
- Is there a row equivalent rectangle?

5.1 and 5.2 interactive video

- Write the definitions in your notes.
- Sketch an input output diagram on the same graph showing a vector sheared horizontally to a vector on a different line that doesn't realign with the original. Label it as NOT an eigenvector.
- Multiply the shear times the x-axis vector
- Write the definition of eigenspace
- Consider the sketch and then open 5152intro.mw in Maple
- Open the geogebra above the video and enter a=0, b=1, c=1 and d=0. Drag around and look for other realignments.
- Compare and contrast the GeoGebra visualization, algebraic representation and Maple output.

- Consider how any reflection about a line in 2-space will have an eigenvalue of 1 corresponding to the line of reflection and an eigenvalue of -1 corresponding to the line perpendicular to the line of reflection.
- Consider a 180-degree rotation in 3-space about the z-axis. Consider what realigns on the same line through the origin to give us eigenvectors (hint consider the z-axis and separately consider a vector in the $x-y-0$ plane—what does the 180 degree rotation do to them)?
- For nontrivial solutions, consider what we can say about the matrix $(A - \lambda I)$
- What is the determinant?
- Write the definitions and consider how we applied them to solve for the eigenvalues
- Consider how I solved for the nullspace of $A - 0I$ and then solve for the nullspace of $A - 1I$ by-hand (i.e. for the eigenspace corresponding to 1)
- What linear transformation is the 2x2 matrix of all 1/2?
- Consider the quadratic formula in action and geometric implications. Then open the geogebra above the video and enter $a = 3, b = -2, c = 1$ and $d = -1$. Drag around and experience the realignments. Then sketch an input output diagram for each eigenvalue.
- Compare your input-output diagrams with mine and consider what are the entire eigenspaces corresponding to each eigenvector?
- Consider how the length of a vector changes as it is scaled by lambda and sketch the input-output vector in your notes, with the axes labeled, and the input and output identified.
- What does a 0 eigenvalue mean?
- Why don't see see an output for vectors on $y = -x$ here?
- Write the characteristic equation (this matrix is NOT triangular so we can't apply that theorem) and solve it. What are the real eigenvalues?
- What linear transformation is this and consider why nothing except the $\vec{0}$ vector realigns
- Find the eigenvalues in Maple or by hand of this stochastic or Markov matrix
- Solve for the nullspace for the eigenvalue of 1?
- That was for the eigenvalue of 1, but the other eigenvalue is .3. What does the .3 mean for vectors in its eigenspace?

5.6 interactive video

- Write the system as a matrix vector equation
- Consider the second row of the coefficient matrix. Can you see why this might make sense in real life?
- What is $A\vec{x}_i$? Apply the definition of eigenvector.
- Open 56intro.mw above the video and execute the commands

- Sketch the trajectory and the two eigenvectors, making sure the trajectory never crosses the line either of the eigenvectors is on (it is between the two in this example)
- Consider which parts of the eigenvector decomposition have k in it and which do not
- Consider that for most initial conditions, since $(\frac{7}{10})^k$ goes to 0 while $(\frac{11}{10})^k$ goes to infinity, the system will tend towards the $y = \frac{5}{4}x$ line corresponding to the larger magnitude eigenvalue
- Fill in the long-term behavior in your notes
- Write out the eigenvector decomposition
- What is the owls and woodrats trajectory in the longterm?
- What ratio do the populations tend to in the longterm for most initial conditions?
- Is $x = 2, y = 5$ also an eigenvector corresponding to $\frac{9}{10}$?
- Sketch a trajectory diagram
- Consider what longterm behavior comes from the dominant eigenvalue and what longterm behavior comes from the dominant eigenvector.
- Notice the 19/240 that we solved for is in Maple. What is the dominant eigenvalue after executing the Eigenvectors command?
- What are the relative populations in the longterm for most initial conditions?
- Sketch a trajectory diagram for an eigenvalue of 1 in your notes.
- Which is true in the long run for this system?
- Consider the trajectory diagram and longterm behavior.
- For the two predators, answer the questions to consider behavior in the longterm, in the limit.
- Consider how we can never cross an eigenspace because $A\vec{x} = \lambda\vec{x}$ tells us we never leave an eigenspace, only scale along them, and that we are linear in the long run, typically asymptotic to the dominant eigenvector.
- Consider how we are always linear in the long run when we have real eigenvalues.

2240 intro interactive video

- What should you call me in all communications?
- open up the course calendar above the video
- Even through there are hard deadlines, where is flexibility built in to the course?
- Is the reading guide all that is required?
- How do we earn completion for interactive videos?
- What are some of the features of the interactive videos?

- What order should items be completed?
- Where do we go to see my feedback on practice quizzes?
- How is the second chance practice quiz after the deadline different then the original (also repeatable!) practice?
- Consider the welcoming environment of this class
- Even with a checkmark for completion, you might have some aspects incorrect on handwrite practice. How can you tell what is incorrect on the handwrite?
- How do we know if you achieved completion on a given activity?
- Are the ASULearn completion activities important?
- There are strict deadlines for handwrites, begins, problem sets, and the final project. The other activities are designed to have you complete them in the order they are listed but also have second chances. Consider the grading, deadlines, and the PDF calendar.
- Consider the general skills from the prerequisite of Calculus with Analytic Geometry I that we need in linear algebra and consider whether there are any you'll want to review (tutoring or online sources are great for review)
- Where is help available outside of class?
- What is some common advice from prior students?
- Our hybrid class is officially designed by the registrar for our third hour to be a part of the activities between classes. The university recommends 2–3 hours outside of class for each credit hour (c.h.): that gives 2*3c.h. + 1 to 3*3c.h. + 1 as weekly engagement time outside of our classes where the +1 is that third hour. Since there are two of those each week, divide these ranges by 2 to see the university recommended hours between each class.

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