# 2.2 Handwrite

**Welcoming Environment**: Actively listen to others and encourage everyone to participate! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Discuss and keep track of any questions your group has. Ask me questions during group work time as well as when I bring us back together. Try to help each other solidify and review the language of linear algebra, algebra, visualizations and intuition from this section, including those related to:

- matrices: invertible (nonsingular) matrix, noninvertible (singular) matrix, elementary matrix
- determinant and inverse of a $2 \times 2$ matrix
- connection between invertibility and unique solutions
- inverse of a product of matrices and inverse of a transpose

Take out your notes from the activities due today as well as the fill-in guide. Use them and each other to respond to the following by handwriting in the language of our class. Use only what we have covered so far in our readings, videos and quizzes.

1. **Building Community**: What are the preferred first names of those sitting near you? If you weren't able to be there, give reference to anyone you had help from or write N/A otherwise.

2. Consider Theorem 5: If $A$ is an invertible $n \times n$ matrix, then for each $\vec{b}$ in $\mathbb{R}^n$, <u>the equation $A\vec{x} = \vec{b}$ has a unique solution $\vec{x} = A^{-1}\vec{b}$</u>

   a) Complete the following: The columns of $A$ are linearly independent if the matrix equation $A\vec{x} = 0$ has only ...

   b) Assume that $A_{n\times n}$ is invertible. First, write this assumption below. Then apply Theorem 5 by substituting in $\vec{b} = \vec{0}$ to the conclusion of Theorem 5 that starts with "the equation..." See the part I underlined above and rewrite that part—you will have substituted in $\vec{b} = \vec{0}$ in two places. Next, name the theorem as your reasoning. Then, simplify the last multiplication by the inverse. Examine this short argument and connect it to the definition of linearly independent you wrote in part a) to show the linear independence of the columns of $A$. Stick to Theorem 5 rather than any material that comes later in the book.

3. Suppose that $A, B$ and $C$ are invertible $n \times n$ matrices.

   a) List a matrix $D$, written using only products of $A, B$ and $C$ or their inverses, so that $(ABC)D = I_{n \times n}$. Write only $D$ in this part, i.e. what is $(ABC)^{-1}$ written out? Hint: In the 2.2 intro video, I mentioned a socks-shoes analogy to help us remember that $(AB)^{-1} = B^{-1}A^{-1}$. Consider the natural extension to 3 matrices.

   b) Using the $D$ you listed in part a), write $(ABC)D$ and substitute your $D$ from part a). Next, apply matrix algebra to reduce. Show all the steps for the matrix algebra.

   c) How many times did we need to use associativity in part b)?

Next, as time allows before I bring us back together, work on the additional activities including any pollev activities and respond in your notes rather than here.

**Help each other and PDF responses to ASULearn**: If you are finished with the handwrite and additional activities before I bring us back together, first ensure that your entire group is finished too, and if not, help each other. Then submit your handwrite, continue reviewing and solidifying or discuss upcoming class work.

Collate your handwritten responses, preferably on this handout, into one full size multipage PDF for submission in the ASULearn assignment. I recommend you turn it in sometime today, but you have until the morning before the next class.