Question **1**

Not complete

Points out of 4.00

Theorem 8 (what makes a matrix invertible) tells us that for an $n \times n$ matrix *A* the following statements are logically equivalent.

(a) *A* is an invertible matrix
(b) *A* is row equivalent to the $n \times n$ identity matrix
(c) *A* has *n* pivot positions
(d) The equation $A\vec{x} = \vec{0}$ has only the trivial solution
(e) The columns of A form a linearly independent set
(g) The equation $A\vec{x} = \vec{b}$ has at least one solution for each $\vec{b}$ in $\mathbb{R}^n$
(h) The columns of A span $\mathbb{R}^n$
(j) There is an $n \times n$ matrix C such that $CA = I$
(k) There is an $n \times n$ matrix D such that $AD = I$
(l) A^T is an invertible matrix

Which statement is the matrix form of the actual definition of the columns of the matrix being linearly independent?

○ *A* has *n* pivot positions

○ The equation $A\vec{x} = \vec{0}$ has only the trivial solution

○ The equation $A\vec{x} = \vec{b}$ has at least one solution for each $\vec{b}$ in $\mathbb{R}^n$

Which statement is the matrix form of the actual definition of the columns of the matrix spanning all of $\mathbb{R}^n$?

○ *A* has *n* pivot positions

○ The equation $A\vec{x} = \vec{0}$ has only the trivial solution

○ The equation $A\vec{x} = \vec{b}$ has at least one solution for each $\vec{b}$ in $\mathbb{R}^n$

If a matrix is square but is NOT row-equivalent to the identity, which are true?
Choose all that apply:

☐ The columns of *A* form a linearly independent set

☐ *A* has fewer than *n* pivot positions

☐ The equation $A\vec{x} = \vec{0}$ has a non-trivial solution, i.e. infinite solutions

☐ *A* has a left inverse but not a right inverse

If a matrix is not square, are the statements in the theorem logically equivalent?

○ yes

○ no

Check

Question **2**

Not complete

Points out of 1.00

The invertible matrix theorem (theorem 8) (what makes a matrix invertible) part (g) says “the equation $A\vec{x} = \vec{b}$ has at least one solution for each $\vec{b}$ in $\mathbb{R}^n$.” However, the note at the bottom of the page says that we could replace the bold, highlighted section of the statement with which of the following?

○ 0 solutions

○ 1 unique solution

○ infinitely many solutions

If A has columns that are linearly independent must A always be invertible?

○ yes

○ not if A is not square

Check

Question **3**

Not complete

Points out of 2.00

Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 3 & 0 & 0 \\ -3 & -4 & 0 \\ 8 & 5 & -3 \end{bmatrix}.$$

Is the matrix invertible?

○ yes

○ no

Which of the following can be expanded to valid arguments here?

○ a) Gaussian reduce to see it has 2 pivot positions

○ b) Gaussian reduce to see it has 3 pivot positions

○ c) A matrix is invertible if and only if its transpose is

○ a) and c)

○ b) and c)

○ none of the above

Check

Question **4**

Not complete

Points out of 1.00

Is the statement "If *A* is an *nxn* matrix then the equation $A\vec{x} = \vec{b}$ has at least one solution for each $\vec{b}$ in $\mathbb{R}^n$" true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

- True and I found a phrase and page number from the text
- False and I can provide a counterexample
- other

Check

Question **5**

Not complete

Points out of 1.00

Is the statement "If the equation $A\vec{x} = \vec{0}$ has a nontrivial solution, then A_{nxn} has fewer than *n* pivot positions" true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

- True and I found a phrase and page number from the text
- False and I can provide a counterexample
- other

Check

Question **6**

Not complete

Points out of 1.00

Is the statement "If the columns of A_{nxn} are linearly independent, then the columns of *A* span $\mathbb{R}^n$ " true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

- True and I found a phrase and page number from the text
- False and I can provide a counterexample
- other

Check

Question **7**

Not complete

Points out of 1.00

Is the statement "If there is a $\vec{b}$ in $\mathbb{R}^n$ such that the equation $A\vec{x} = \vec{b}$ is consistent, where *A* is *nxn*, then the solution is unique" true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

○True and I found a phrase and page number from the text

○False and I can provide a counterexample

○other

Check

Question **8**

Not complete

Points out of 1.00

If the equation $C\vec{u} = \vec{v}$ has more than one solution for some $\vec{v}$ in $\mathbb{R}^n$, can the columns of the *nxn* matrix *C* span $\mathbb{R}^n$? Why or why not?

○$C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ shows that it is possible since $C\vec{u} = \vec{0}$ has infinitely many solutions and the columns of *C* span $\mathbb{R}^2$

○the columns can span by Theorem 8 (inverse matrix theorem)

○both of the above

○the columns can not span, by Theorem 8 (inverse matrix theorem)

Check

Question **9**

Not complete

Points out of 11.00

Consider the nonsquare matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and reduce it using strict Gaussian to row echelon form (1 step).

What is the pivot for *y*, the second column, if any?

- ○ *y* has no pivot
- ○ 1
- ○ -2
- ○ -3
- ○ other

Does *A* have a pivot for every column?

- ○ yes
- ○ no

Does *A* have linearly independent columns?

- ○ yes
- ○ no

Does *A* have a pivot in every row?

- ○ yes
- ○ no

Do the columns of *A* span all of $\mathbb{R}^2$?

- ○ yes
- ○ no

Consider the nonsquare matrix $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ and reduce it using strict Gaussian to row echelon form (3 steps).

What is the pivot for *y*, the second column, if any?

- ○ *y* has no pivot
- ○ 1
- ○ -2
- ○ -3
- ○ other

Does *B* have a pivot for every column?

- yes
- no

Does *B* have linearly independent columns?

- yes
- no

Does *B* have a pivot in every row?

- yes
- no

Do the columns of *B* span all of $\mathbb{R}^3$?

- yes
- no

Are either of these matrices invertible?

- yes
- no

Check

Question **10**

Not complete

Points out of 3.00

Assume that *F* is an *nxn* matrix. If the equation $F\vec{x} = \vec{y}$ is inconsistent for some $\vec{y}$ in $\mathbb{R}^n$, what can you say about the equation $F\vec{x} = \vec{0}$? Why? What happens when *F* is not square?

When *F* is an *nxn* matrix and $F\vec{x} = \vec{y}$ is inconsistent for some $\vec{y}$ in $\mathbb{R}^n$, then

○ $F\vec{x} = \vec{0}$ is inconsistent

○ $F\vec{x} = \vec{0}$ has only the trivial solution

○ $F\vec{x} = \vec{0}$ has infinitely many solutions

Can we use Theorem 8 (inverse matrix theorem) to show why?

○ yes

○ no

What happens when *F* is not square?

○ a) If *F* has fewer columns than rows, with $F\vec{x} = \vec{y}$ inconsistent for some *y*, then $F\vec{x} = \vec{0}$ can have one solution

○ b) If *F* has more columns than rows, with $F\vec{x} = \vec{y}$ inconsistent for some *y*, then $F\vec{x} = \vec{0}$ must have infinite solutions

○ c) both a) and b)

○ none of the above

Check

Question **11**

Not complete

Points out of 1.00

Suppose we are working with the matrix *A* that is invertible (non-singular) but with a high condition number. We conclude that this matrix is ill-conditioned. What does this mean?

○ An ill-conditioned matrix is an invertible matrix for which changing its entries very slightly can make the matrix no longer invertible.

○ An ill-conditioned matrix is one with particularly large entries that cause storage issues in computation

○ An ill-conditioned matrix is one that originally contains many decimal entries

○ all of the above

Check

Question **12**

Not complete

Points out of 1.00

To solidify and prepare for upcoming work, review and contemplate your knowledge and any questions that remain as related to definitions, concepts, computations, and examples from 2.3, including

- what makes a matrix invertible for a square matrix (Theorem 8 statements aside from f. and i., which we haven't covered)
- condition number (numerical note on p. 123)

and consider 2.2, including

- matrices: invertible (nonsingular) matrix, noninvertible (singular) matrix, elementary matrix
- determinant and inverse of a 2x2 matrix
- connection between invertibility and unique solutions
- inverse of a product of matrices and inverse of a transpose

and 2.1, including

- matrices: diagonal matrix [and main diagonal], zero matrix
- matrix operations: matrix addition, scalar multiplication of a matrix, matrix multiplication, powers of a matrix, left (or right) multiplication, transpose of a matrix
- matrix multiplication by linear combinations of the columns of A using weights from the corresponding column of B or by the dot products of a row of A with the corresponding column of B.
- algebraic properties that do hold for matrix multiplication: associativity and one-sided distributivity
- algebraic properties that don't hold for matrix multiplication: commutativity

Since the material builds on itself, consider whether there is any material from before this module that you want to brush up on:

1.1

- algebra of linear equations: coefficients and variables
- geometry of linear equations in 2D and 3D: lines and planes
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions
- matrix of a linear system: coefficient matrix, augmented matrix, triangular form
- row equivalent systems
- algorithm for solving a linear system using elementary row operations of replacement, interchange, and scaling

1.2

- matrix of a linear system: row echelon form (Gaussian), reduced row echelon form (Gauss-Jordan)
- pivots: pivot position of a matrix, pivot column of a matrix
- row reduction algorithm we will most commonly use: elimination by forward phase and back substitution to row echelon form
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions with free variables and parametric solutions

1.3

- algebra of vectors: coordinates, addition of vectors, scalar multiplication of vectors, properties like associativity under addition (property ii on p. 29), a linear combination with weights, zero vector, span of a set of vectors=all the linear combinations, is a vector in the span?, vector equation $\longrightarrow$ augmented matrix
- geometry of vectors in 2D and 3D: directed segment, parallelogram for addition, on same line for scalar multiplication of vectors, origin=zero vector, a linear combination geometrically in the plane or 3D, span=all the linear combinations geometrically in the plane or 3D, spaces of subsets of R^n spanned by vectors

1.4

- algebra of matrix vector equation $A\vec{x} = \vec{b}$:
  - multiply a matrix and a column vector by linear combinations of the columns of *A* using weights from $\vec{x}$
  - span of the columns of *A* = set of all linear combinations of the columns of *A*
  - matrix vector equation $\longrightarrow$ vector equation $\longrightarrow$ augmented matrix
  - equations generic vectors $\vec{b}$ must satisfy to be in the span (Example 3)
  - dot products of rows of *A* with $\vec{x}$,
- geometry of solutions of matrix vector equation $A\vec{x} = \vec{b}$: spaces of subsets of R^3 spanned by the column vectors of A, geometry of such spaces (Figure 1)

- Theorem 4: relationship of consistency of $A\vec{x} = \vec{b}$ to always being a linear combination to spanning the entire R^m, where *m* is the number of rows, to having a pivot position in every row of *A*.
- identity matrix *I*

1.5

- algebra of homogeneous systems: $A\vec{x} = \vec{0}$
- algebraic solutions of homogenous systems always include the trivial solution=$\vec{0}$. nontrivial solutions, if any exist, are parameterized in parametric vector form using free variables to express those as well as the variables with pivots and then decomposed algebraically to showcase the algebra and geometry giving $t\vec{v}$ or $s\vec{u} + t\vec{v}$ or similar, where each free variable is attached to a vector.
- geometry of solutions of homogeneous systems are geometric spaces through the origin like lines, planes, or hyperplanes
- algebra of nonhomogeneous systens: $A\vec{x} = \vec{b}$
- solutions of nonhomogeneous systems in parametric vector form can be decomposed algebraically to showcase the algebra and geometry like $\vec{p} + t\vec{v}$, vectors ending on the line parallel to $\vec{v}$ or $\vec{p} + s\vec{u} + t\vec{v}$, vectors ending on the plane parallel to the one spanned by $\vec{u}, \vec{v}$...
- geometry of solutions of nonhomogeneous systems are geometric spaces translated away from the origin via adding $\vec{p}$

1.7

- linearly independent set of vectors and connection to a homogeneous equation having only the trivial solution
- linearly dependent set of vectors and connection to nontrivial solutions existing and providing a dependance relation
- geometry of linearly independent set of 2 vectors: independent directions in space versus along the same line (Figure 1)
- geometry of linearly independent set of 3 or more vectors: no one vector is in the span of the rest, i.e. they are all needed to span the space versus redundancy in the geometric space they span in the sense that they aren't all needed to generate the same space under linear combinations (Figure 2)
- linearly independent columns of a matrix
- redundancy of $\vec{0}$ in a set of vectors $\{\vec{v}_1 = \vec{0}, \vec{v}_2, \ldots \vec{v}_n\}$ (Theorem 9)

When you have finished reviewing and reflecting, select one of the following (both receive full credit)

○I currently have no questions

○I will continue solidifying and understand that help is available in Dr. Sarah's more extensive feedback that follows below each question after I finish and open back up an entire practice quiz (this is more extensive than the hints that I can access during the open quiz), in Dr. Sarah's glossary/Wiki which is embedded into ASULearn from the linked terms, in Dr. Sarah's office hours and forum, and in Math Lab and Tutoring

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