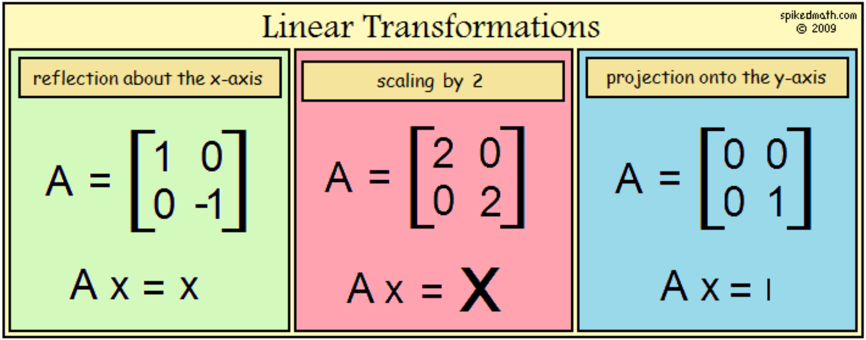
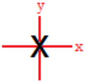
Applications of Linear Transformations



http://spikedmath.com/comics/109-linear-transformations.png



Review of Dot Products for Matrix Multiplication

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix}$$

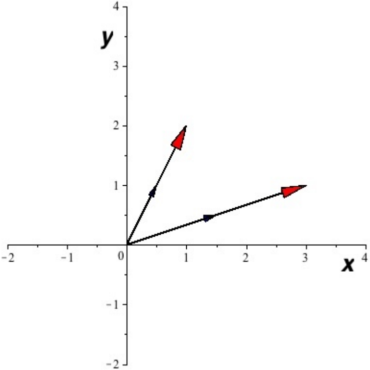
$$= \begin{bmatrix} \begin{bmatrix} \text{row } 1A \end{bmatrix} \cdot \begin{bmatrix} \\ \text{col } 1B \\ \\ \end{bmatrix} & \begin{bmatrix} \text{row } 1A \end{bmatrix} \cdot \begin{bmatrix} \\ \text{col } 2B \\ \\ \end{bmatrix} \\ \begin{bmatrix} \text{row } 2A \end{bmatrix} \cdot \begin{bmatrix} \\ \text{col } 1B \\ \\ \end{bmatrix} & \begin{bmatrix} \text{row } 2A \end{bmatrix} \cdot \begin{bmatrix} \\ \text{col } 2B \\ \\ \end{bmatrix} \end{bmatrix}$$

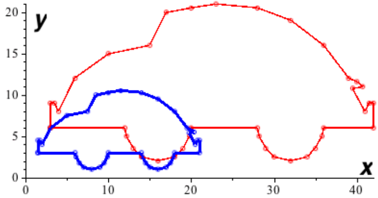
$$= \begin{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 9 \\ 11 \end{bmatrix} & \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 10 \\ 12 \end{bmatrix} \\ \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 9 \\ 11 \end{bmatrix} & \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 10 \\ 12 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 1 \cdot 7 + 2 \cdot 9 + 3 \cdot 11 & 1 \cdot 8 + 2 \cdot 10 + 3 \cdot 12 \\ 4 \cdot 7 + 5 \cdot 9 + 6 \cdot 11 & 4 \cdot 8 + 5 \cdot 10 + 6 \cdot 12 \end{bmatrix}$$

Input thinner red and output thicker blue.

$$\text{Car}=\begin{bmatrix} 5 & 2 & 2 & 5 & 5 & 5 & 2 & 2 & 3.5 & 3.5 & 2 & 2 & 5 & 3.5 & 3.5 & 5 & 5 & 5 & 2 & 2 & 5 \\ 2 & 2 & 2 & 2 & 2 & 8 & 8 & 8 & 7 & 3 & 2 & 8 & 8 & 7 & 3 & 2 & 8 & 8 & 8 & 2 & 2 \end{bmatrix}$$





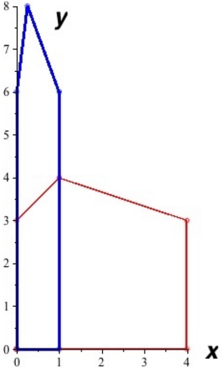
Images created using VLA Package from *Visual Linear Algebra* by Herman and Pepe

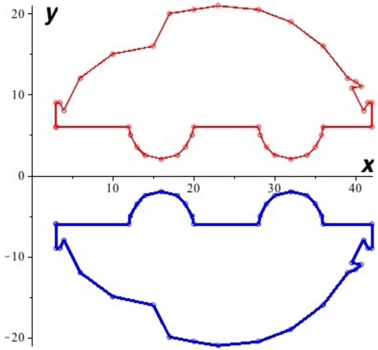
Output of unit axes are the columns:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ c \end{bmatrix} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} b \\ d \end{bmatrix}$$

$$\text{Dilation Car} = \begin{bmatrix} .5 & 0 \\ 0 & .5 \end{bmatrix} \text{Car}$$

Input thinner red and output thicker blue.





Images created using VLA Package from *Visual Linear Algebra* by Herman and Pepe

$\begin{bmatrix} .25 & 0 \\ 0 & 2 \end{bmatrix}$ House

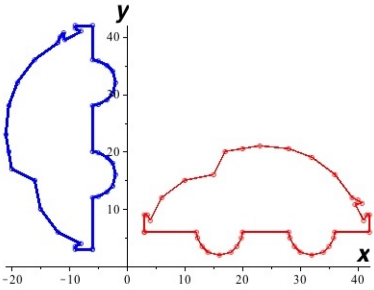
mixed dilation

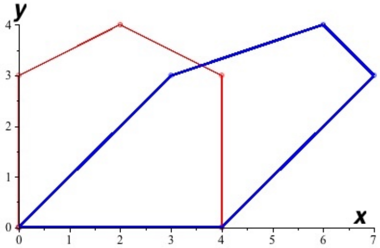
$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ Car

reflection

Math isn't important for programming
and other hilarious jokes you can tell yourself

Input red and output blue. What are the names and matrices?





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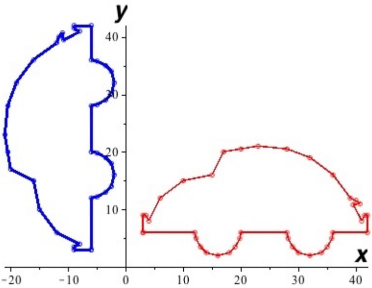
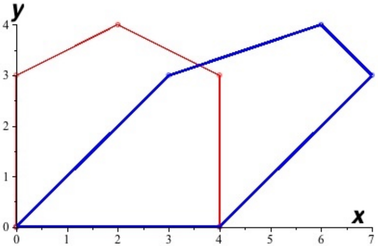




Images created using VLA Package from *Visual Linear Algebra* by Herman and Pepe

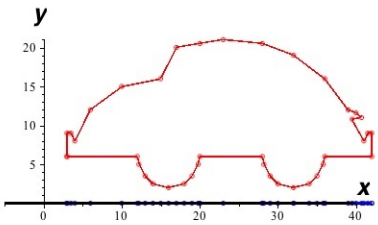
sheep

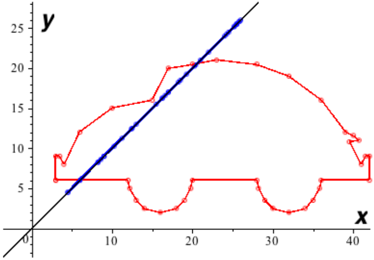
sheared sheep

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$ Car

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$ House

Input red and output blue.
What are the lines of projections and matrices?





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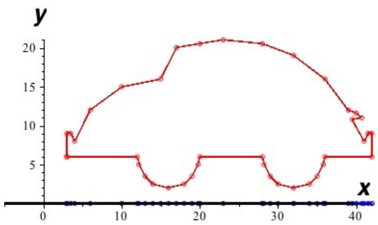
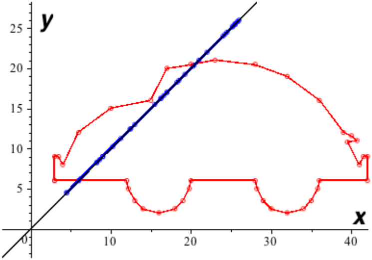


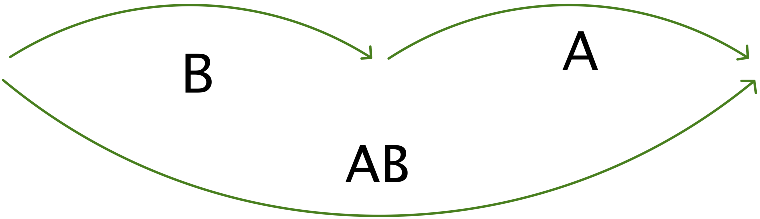


Images created using VLA Package from *Visual Linear Algebra* by Herman and Pepe

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ Car

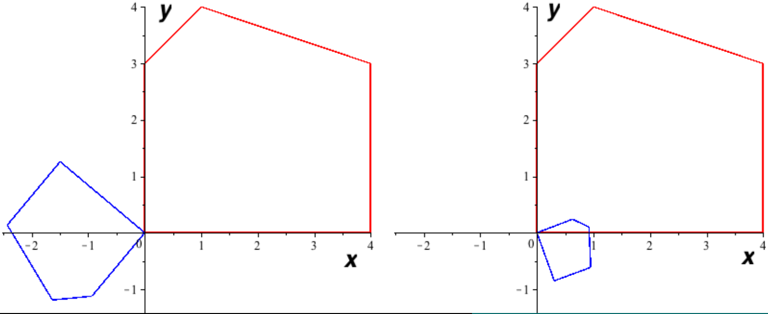
$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ Car

Composite Transformations



ABHouse

Movie created using VLA Package from *Visual Linear Algebra* by Herman and Pepe



Transformations of $\mathbb{R}^2$

Rotation Matrix: $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$

Dilation Matrix: $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

Horizontal Shear: $\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$ Vertical Shear: $\begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$

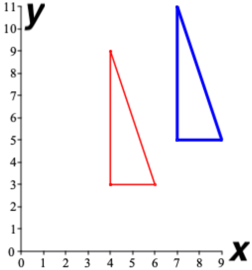
Projection onto $y = x$ $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

Reflection across the y-axis $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

- What important transformation is missing?

Homogeneous Coordinates to Move Origin

$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \neq \begin{bmatrix} x+3 \\ y+2 \end{bmatrix} \qquad \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$



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Homogeneous Coordinates to Move Origin

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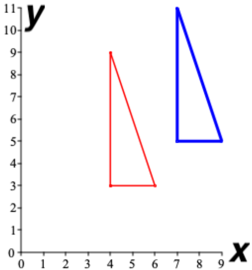

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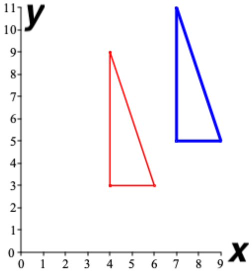
translation matrix using $z = 1$ plane

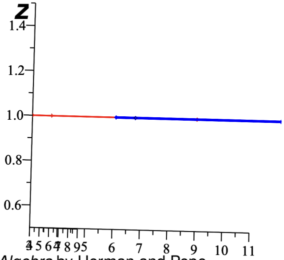
$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \cdot x + 0 \cdot y + 3 \cdot 1 \\ 0 \cdot x + 1 \cdot y + 2 \cdot 1 \\ 0 \cdot x + 0 \cdot y + 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} x+3 \\ y+2 \\ 1 \end{bmatrix}$$

Translated Triangle

translation: $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ triangle: $\begin{bmatrix} 4 & 4 & 6 & 4 \\ 3 & 9 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 4 & 6 & 4 \\ 3 & 9 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 4+3 & 4+3 & 6+3 & 4+3 \\ 3+2 & 9+2 & 3+2 & 3+2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$





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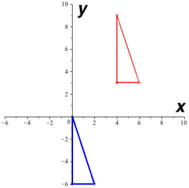
Rotate a Triangle About $\begin{bmatrix} 4 \\ 9 \end{bmatrix}$

Step 1. Translate the triangle so the top vertex $\begin{bmatrix} 4 \\ 9 \end{bmatrix}$ is at $\vec{0}$.

Step 2. Rotate 45° about $\vec{0}$ in homogeneous coordinates.

Step 3. Translate back from $\vec{0}$ to where we started at $\begin{bmatrix} 4 \\ 9 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\frac{\pi}{4}) & -\sin(\frac{\pi}{4}) & 0 \\ \sin(\frac{\pi}{4}) & \cos(\frac{\pi}{4}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -9 \\ 0 & 0 & 1 \end{bmatrix} \text{Triangle}$$



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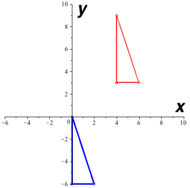

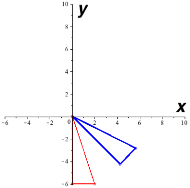

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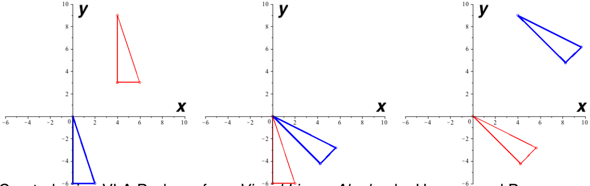
$$\begin{bmatrix}1 & 0 & 4\\0 & 1 & 9\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos(\frac{\pi}{4}) & -\sin(\frac{\pi}{4}) & 0\\ \sin(\frac{\pi}{4}) & \cos(\frac{\pi}{4}) & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & -4\\0 & 1 & -9\\0 & 0 & 1\end{bmatrix}\text{Triangle}$$



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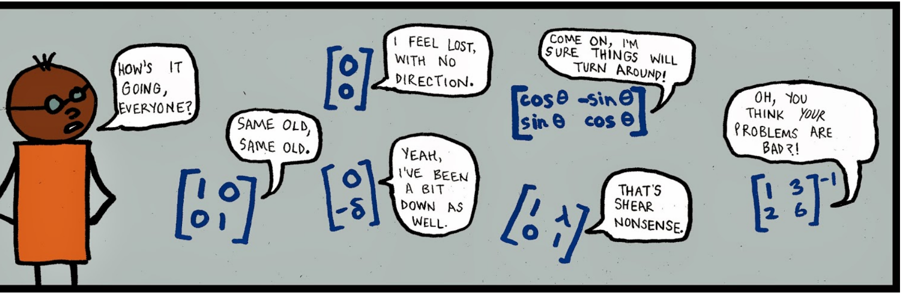
Which of the following are true about linear transformations?

a) points go in as column vectors, and the first column of the transformation represents the output of the unit x-axis
b) we must use homogeneous coordinates (higher dimensional coordinate=1) if we want to move the origin
c) they compose like functions with the first transformation on the far right, next to the digital image
d) all of the above
e) two of the above



https://pbs.twimg.com/media/BhKmT5RIYAA4iU6.jpg:large

To rotate a figure $\begin{bmatrix} x_1 & \dots & x_p \\ y_1 & \dots & y_p \\ 1 & \dots & 1 \end{bmatrix}$ about a point (-2,3) we can:

a) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 & \dots & x_p \\ y_1 & \dots & y_p \\ 1 & \dots & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 & \dots & x_p \\ y_1 & \dots & y_p \\ 1 & \dots & 1 \end{bmatrix}$

c) $\begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 & \dots & x_p \\ y_1 & \dots & y_p \\ 1 & \dots & 1 \end{bmatrix}$

d) more than one of the above

e) none of the above

Making a Car Movie

How can we keep a car on a road $(\cos(t), \sin(2t))$?

Making a Car Movie

How can we keep a car on a road $(\cos(t), \sin(2t))$?

$$\begin{bmatrix} 1 & 0 & \cos(t) \\ 0 & 1 & \sin(2t) \\ 0 & 0 & 1 \end{bmatrix} \text{car}$$

Making a Car Movie

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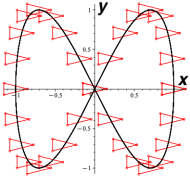

VLA Package from *Visual Linear Algebra* by Herman and Pepe

How can we point the car in the direction of motion?

Making a Car Movie

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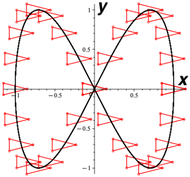


VLA Package from *Visual Linear Algebra* by Herman and Pepe

How can we point the car in the direction of motion?

$$\begin{bmatrix} \frac{-\sin(t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & \frac{-2\cos(2t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & 0 \\ \frac{2\cos(2t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & \frac{-\sin(t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ car}$$

Order and Unit Norm Matters!

Car on the road and pointed in the direction of motion:

$$\begin{bmatrix} 1 & 0 & \cos(t) \\ 0 & 1 & \sin(2t) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{-\sin(t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & \frac{-2\cos(2t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & 0 \\ \frac{2\cos(2t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & \frac{-\sin(t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & 0 \\ 0 & 0 & 1 \end{bmatrix} car$$

Incorrect:

$$\begin{bmatrix} \frac{-\sin(t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & \frac{-2\cos(2t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & 0 \\ \frac{2\cos(2t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & \frac{-\sin(t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \cos(t) \\ 0 & 1 & \sin(2t) \\ 0 & 0 & 1 \end{bmatrix} \text{car}$$

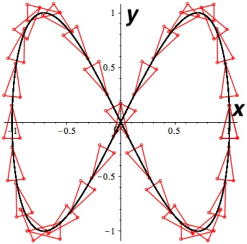
Incorrect:

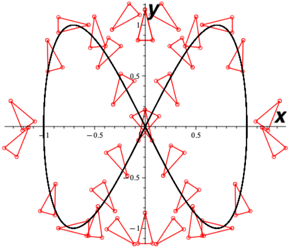
$$\begin{bmatrix} 1 & 0 & \cos(t) \\ 0 & 1 & \sin(2t) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\sin(t) & -2\cos(2t) & 0 \\ 2\cos(2t) & -\sin(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{car}$$

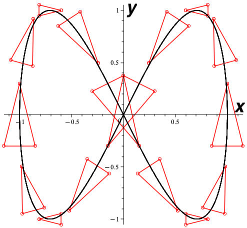
VLA Package from *Visual Linear Algebra* by Herman and Pepe

To turn a car so that it points in the direction of motion, we

a) define a unit vector in the direction of the velocity (tangent) of the curve by dividing by its length/norm so it won't change the size of the car
b) create an orthogonal vector to pair with it in a rotation matrix by creating a vector on a line with negative reciprocal slope (swap *x* and *y* and introduce a negative sign)
c) both of the above







VLA Package from *Visual Linear Algebra* by Herman and Pepe

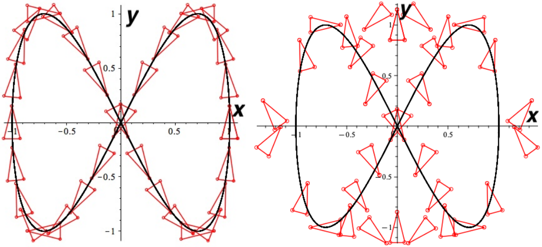
To keep a car on a curved race track, we can perform the appropriate matrix operations in the following order

a) $\begin{bmatrix} 1 & 0 & \cos(t) \\ 0 & 1 & \sin(2t) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{-\sin(t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & \frac{-2\cos(2t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & 0 \\ \frac{2\cos(2t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & \frac{-\sin(t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ car

b) $\begin{bmatrix} \frac{-\sin(t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & \frac{-2\cos(2t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & 0 \\ \frac{2\cos(2t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & \frac{-\sin(t)}{\sqrt{\sin^2(t)+4cos^2(2t)}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \cos(t) \\ 0 & 1 & \sin(2t) \\ 0 & 0 & 1 \end{bmatrix}$ car

c) both of the above

d) none of the above



VLA Package from *Visual Linear Algebra* by Herman and Pepe

Linear Transformations of $\mathbb{R}^3$ in Homogeneous Coords

$$\begin{bmatrix} \cos(\frac{\pi}{6}) & -\sin(\frac{\pi}{6}) & 0 & 0 \\ \sin(\frac{\pi}{6}) & \cos(\frac{\pi}{6}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{spaceship}$$

Created using VLA Package from *Visual Linear Algebra* by Herman and Pepe

Yoda from Star Wars

data from Kecskeméti B. Zoltán (Lucasfilm LTD)

$$1 \text{ of } 4667 \text{ in file } 1: \begin{bmatrix} -1.355884 & 12.452657 & 0.927687 \\ -1.355386 & 12.449140 & 0.928006 \\ -1.356376 & 12.450524 & 0.924873 \end{bmatrix}$$

Yoda from Star Wars

data from Kecskeméti B. Zoltán (Lucasfilm LTD)

1 of 4667 in file 1: $\begin{bmatrix} -1.355884 & 12.452657 & 0.927687 \\ -1.355386 & 12.449140 & 0.928006 \\ -1.356376 & 12.450524 & 0.924873 \end{bmatrix}$

4667 triangles

1 of 29460 in file 2: $\begin{bmatrix} -1.355386 & 12.449140 & 0.928006 \\ -1.355884 & 12.452657 & 0.927687 \\ -1.530717 & 12.421906 & 0.980687 \\ -1.529826 & 12.418444 & 0.980901 \end{bmatrix}$

29460 quadrilaterals

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$(Rot.Yoda^T)^T = Yoda.Rot^T$ via $(AB)^T = B^T A^T$

$$\begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{bmatrix}^T =$$

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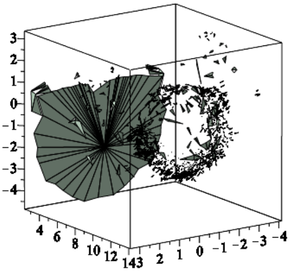
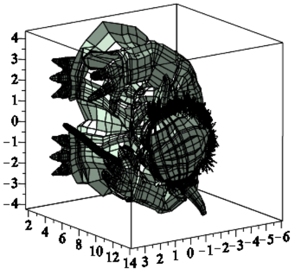
Yoda



YodaRotated



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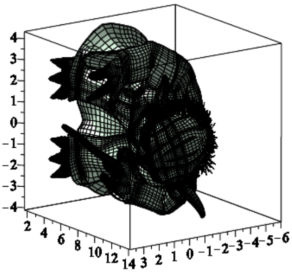

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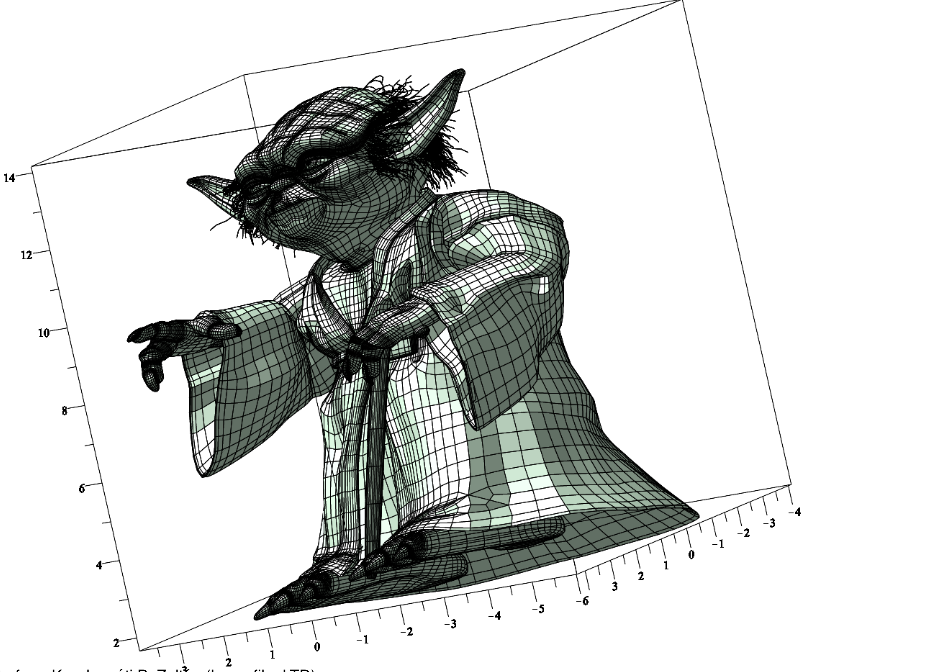

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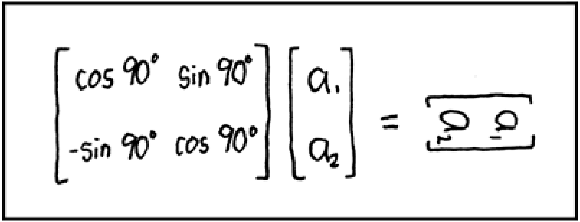
To rotate Yoda, who was given in row vectors as opposed to column vectors, we made use of

a) a matrix that rotates about a line in 3-space

b) $(AB)^T = B^T A^T$

c) both of the above

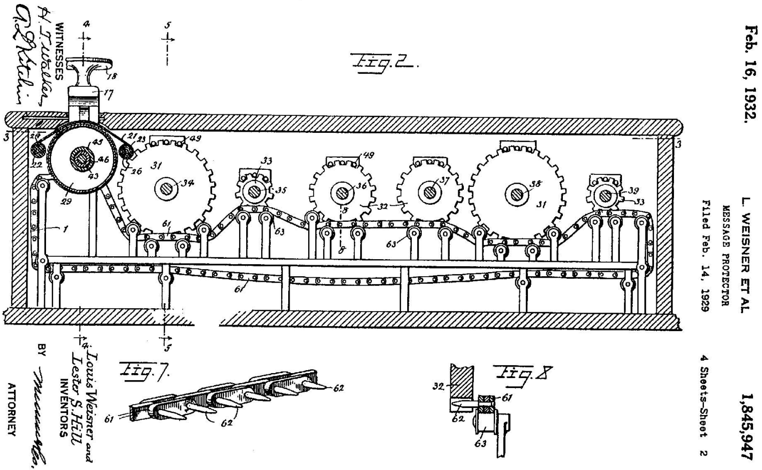
d) none of the above



https://xkcd.com/184/

Mechanical Message Protector

- Louis Weisner & Lester Hill
- U.S. patent number 1845947 from 1929



mechanical device: 6×6 matrix

Hill Cipher

- linear transformation of the digitized alphabet

space	A	B	C	D	E	F	G	$\cdots$	Y	Z
0	1	2	3	4	5	6	7	$\cdots$	25	26

- A[uncoded message] $=$ [coded message]

$$A \begin{bmatrix} \\ \text{col1=start of message} & \dots & \text{coln=end of message} \\ \\ \end{bmatrix}$$

Hill Cipher

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- A[uncoded message] $=$ [coded message]

$$A\left[\begin{array}{ccc} \text{col1=start of message} & \ldots & \text{coln=end of message} \end{array}\right]$$

- matrix algebra
  $A^{-1}(A[\text{uncoded message}]) = A^{-1}[\text{coded message}]$
  $(A^{-1}A)[\text{uncoded message}] = A^{-1}[\text{coded message}]$
  $I\,[\text{uncoded message}] = A^{-1}[\text{coded message}]$
  $[\text{uncoded message}] = A^{-1}[\text{coded message}]$
- method is vulnerable to those that intercept enough vector correspondences (uncoded and coded) because of its linearity.

35, 10, 42, 21, 5, 2, 28, 14, 17, -2, 3, 0, 5, 0, 8, 4, 11, 3, 31, 13

the last 4 numbers decode to HERE

2x2 matrix used in the Hill Cipher

35, 10, 42, 21, 5, 2, 28, 14, 17, -2, 3, 0, 5, 0, 8, 4, 11, 3, 31, 13

the last 4 numbers decode to HERE

2x2 matrix used in the Hill Cipher

11, 3, 31, 13 decodes to HERE, digitized as H=8 E=5 R=18 E=5

Decode $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 11 \\ 3 \end{bmatrix} = \begin{bmatrix} 11a + 3b \\ 11c + 3d \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 31 \\ 13 \end{bmatrix} = \begin{bmatrix} 31a + 13b \\ 31c + 13d \end{bmatrix} = \begin{bmatrix} 18 \\ 5 \end{bmatrix}$$

$$\begin{aligned} 11a + 3b &= 8 \\ 11c + 3d &= 5 \\ 31a + 13b &= 18 \\ 31c + 13d &= 5 \end{aligned}$$

$$\begin{array}{c} \begin{array}{ccccc} a & b & c & d & = \end{array} \\ \begin{bmatrix} 11 & 3 & 0 & 0 & 8 \\ 0 & 0 & 11 & 3 & 5 \\ 31 & 13 & 0 & 0 & 18 \\ 0 & 0 & 31 & 13 & 5 \end{bmatrix} \end{array} \quad rref$$

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2x2 matrix used in the Hill Cipher
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$$\begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 35 & 42 & 5 & 28 & 17 & 3 & 5 & 8 & 11 & 31 \\ 10 & 21 & 2 & 14 & -2 & 0 & 0 & 4 & 3 & 13 \end{bmatrix}$$

$$\begin{bmatrix} 25 & 21 & 3 & 14 & 19 & 3 & 5 & 4 & 8 & 18 \\ 15 & 0 & 1 & 0 & 21 & 3 & 5 & 0 & 5 & 5 \end{bmatrix}$$

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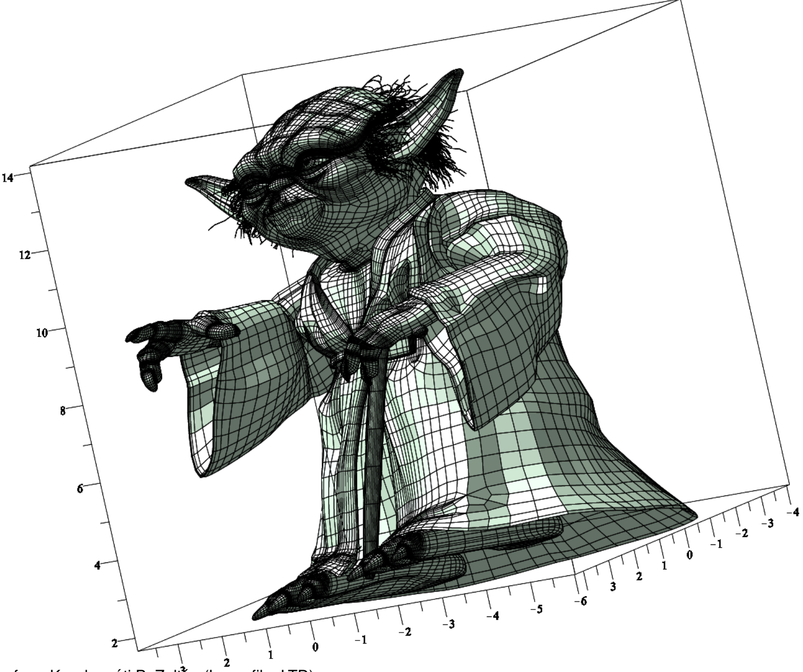
you can succeed here



data from Kecskeméti B. Zoltán (Lucasfilm LTD)