You can preview this quiz, but if this were a real attempt, you would be blocked because:

This quiz is not currently available

Question **1**

Not complete

Points out of 2.00

Why do we need homogeneous coordinates?

- ○ to solve homogeneous equations
- ○ to be able to translate

How does composition of linear transformations work for AB Digital Image?

- ○ first we multiply the digital image by *A*, acting via left multiplication, and then *B*
- ○ first we multiply the digital image by *B*, acting via left multiplication, and then *A*.
- ○ they both work since matrix multiplication is commutative

Check

Question **2**

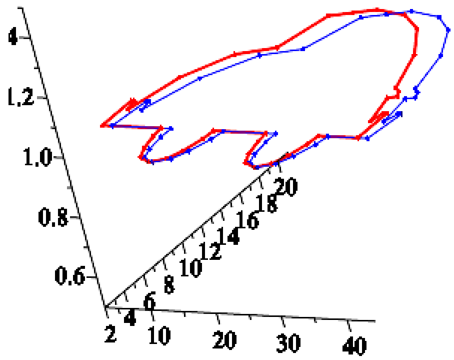
Not complete

Points out of 9.00

What *3x3* matrix *A* will have the same effect in homogeneous coordinates on $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ via $A\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

that the following shear matrix $\begin{bmatrix} 1 & .25 \\ 0 & 1 \end{bmatrix}$ has on $\begin{bmatrix} x \\ y \end{bmatrix}$ via $\begin{bmatrix} 1 & .25 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$?

Here is the shear in homogeneous coordinates acting on a car in Maple (input the thicker red car and output the blue car)



What is the matrix representation of the shear?

Check

Question **3**

Not complete

Points out of 1.00

Find the *3x3* matrices that produce the described composite transformations, using homogeneous coordinates:

Reflect points through the *x*-axis and then rotate 45 degrees counterclockwise about the origin.

○ $\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

○ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

○ both of the above

○ other

Check

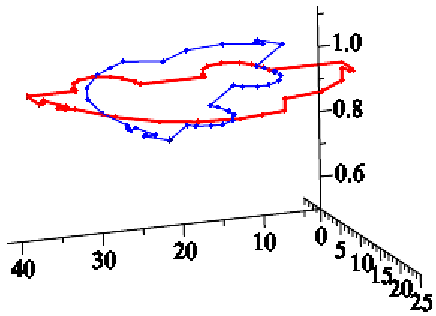
Question **4**

Not complete

Points out of 1.00

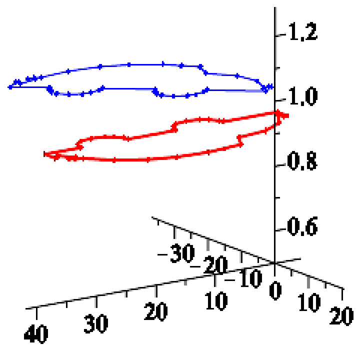
In the last problem, we reflected points through the x-axis and then rotated 45 degrees counterclockwise about the origin. We can act on a homogeneous car in 3-space via $\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$car

A visualization of this, where the thinner blue is the transformed car, is



If we rotate and then reflect the car, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$car

we see



What do these show?

○ matrix multiplication is not commutative so the first transformation must be right next to the car

○ matrix multiplication is not associative

○ both of the above

○ other

Check

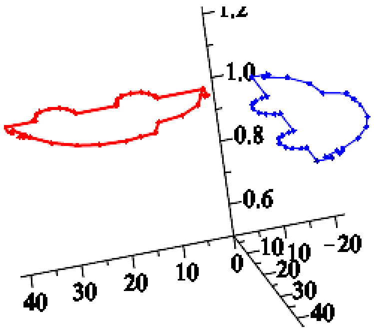
Question **5**

Not complete

Points out of 18.00

Find the *3x3* matrices that produce the described composite linear transformations, using homogeneous coordinates:

Translate by (2,1) and then rotate 90 degrees counterclockwise about the origin. Here is a visualization of this composition acting on a car in Maple:



Write the individual matrices in the correct order for matrix multiplication as a linear transformation $A \times B$, but simplify your responses by entering only integers, with no extra characters or spaces.

×

Check

Question **6**

Not complete

Points out of 4.00

Let *A* and *B* be *2x2* matrices and *D* be a *2x100* matrix.

How many multiplications of numbers do we have in various [matrix multiplication]s?

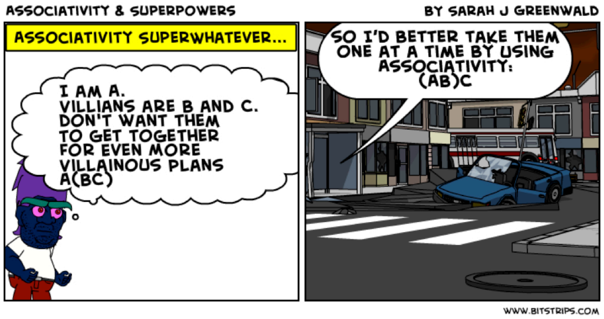
For an expression *ae + bg,* count it as 2 numerical multiplications, 1 for *ae* and the other for *bg*.

How many multiplications are in *AB*?

How many multiplications are in a 2x2 matrix times *D*?

How many multiplications are there from start to finish in *(AB)D*?

How many multiplication are there from start to finish in *A(BD)*?



Check

Question **7**

Not complete

Points out of 3.00

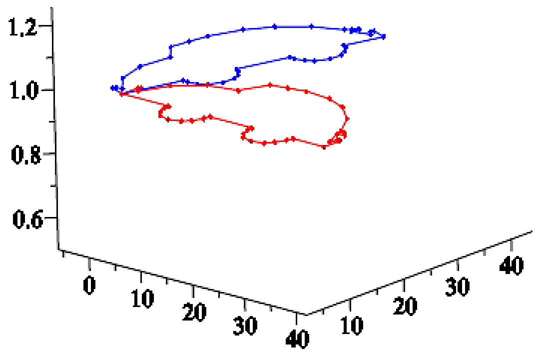
To rotate a car $72^{\circ} = \frac{2\pi}{5}$ about a point other than the origin, we perform matrix3.matrix2.matrix1.car (matrix multiplication in Maple is a period). What are these matrices, from among the following?

matrix a: $\begin{bmatrix} \cos(\frac{2\pi}{5}) & -\sin(\frac{2\pi}{5}) \\ \sin(\frac{2\pi}{5}) & \cos(\frac{2\pi}{5}) \end{bmatrix}$

matrix b: $\begin{bmatrix} \cos(\frac{2\pi}{5}) & -\sin(\frac{2\pi}{5}) & 0 \\ \sin(\frac{2\pi}{5}) & \cos(\frac{2\pi}{5}) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

matrix c: $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$

matrix d: $\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$



To rotate a figure about a point other than the origin, matrix 1 in matrix3.matrix2.matrix1.car is

- ○matrix a
- ○matrix b
- ○matrix c
- ○matrix d

To rotate a figure about a point other than the origin, matrix 2 in matrix3.matrix2.matrix1.car is

- ○matrix a
- ○matrix b
- ○matrix c
- ○matrix d

To rotate a figure about a point other than the origin, matrix 3 in matrix3.matrix2.matrix1.car is

- ○matrix a

- ○ matrix b
- ○ matrix c
- ○ matrix d

Check

Question **8**

Not complete

Points out of 2.00

In the last question, why didn't we use one of the 4 matrices from among

matrix a: $\begin{bmatrix} \cos(\frac{2\pi}{5}) & -\sin(\frac{2\pi}{5}) \\ \sin(\frac{2\pi}{5}) & \cos(\frac{2\pi}{5}) \end{bmatrix}$

matrix b: $\begin{bmatrix} \cos(\frac{2\pi}{5}) & -\sin(\frac{2\pi}{5}) & 0 \\ \sin(\frac{2\pi}{5}) & \cos(\frac{2\pi}{5}) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

matrix c: $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$

matrix d: $\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$

to rotate a figure about a point other than the origin?

- ○since we were making use of translation, which needs a $3x3$ matrix, then we need all our matrices to be 3×3 matrices in homogeneous coordinates because a 3×3 matrix and a 2×2 matrix don't multiply
- ○since we were making use of translation by (3,6), we didn't need the translation by (-3,-6)
- ○both of the above
- ○none of the above

In Maple, I perform the composition of the 3 matrices applied to the car as follows:

with(LinearAlgebra): with(plots): interface(rtablesize=40):

carh:=Matrix([[3, 3, 3.5, 4, 6, 10, 15, 17, 20, 23, 28, 32, 36, 39, 40, 40.7, 39.5, 41, 41.5, 42, 42, 36, 35.8, 35, 34, 32, 30, 29, 28.2, 28, 20, 19.8, 19, 18, 16, 14, 13, 12.2, 12, 3], [6, 9, 9, 8, 12, 15, 16, 20, 20.5, 21, 20.5, 19, 16, 12, 11.6, 11, 10.8, 8, 9, 9, 6, 6, 5, 3.5, 2.5, 2, 2.5, 3.5, 5, 6, 6, 5, 3.5, 2.5, 2, 2.5, 3.5, 5, 6, 6], [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]]);

Translation1:=Matrix([[1,0,-3],[0,1,-6],[0,0,1]]);

Rotation:=Matrix([[cos(2*Pi/5),-sin(2*Pi/5),0],[sin(2*Pi/5),cos(2*Pi/5),0],[0,0,1]]);

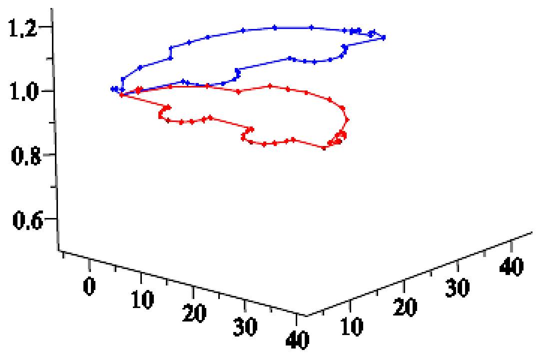
Translation2:=Matrix([[1,0,3],[0,1,6],[0,0,1]]);

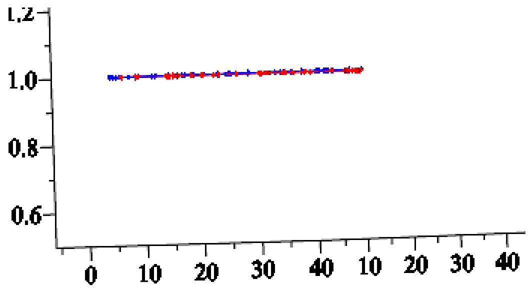
plota1:=pointplot3d(carh, style=pointline, color="Red"):

plota2:=pointplot3d(Translation2.Rotation.Translation.carh, style=pointline, color="Blue"):

display(plota1,plota2);

Here are two views of the result





What does the second graph show?

- ○ that we've done a projection and squished the car to a line
- ○ that they lie in the $z = 1$ plane in 3-space as any in homogeneous coordinate will do
- ○ both of the above

Check

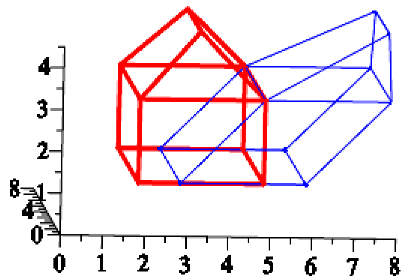
Question 9

Not complete

Points out of 4.00

Guess the Transformation 1

Here we see an input as the red house (thicker lines) in 3-space and output as the blue one that I created in Maple.

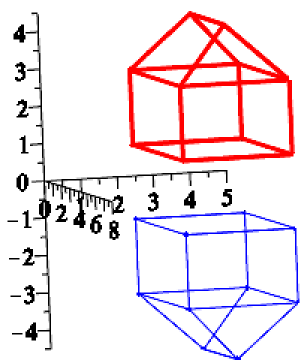


The x-axis is the one on the bottom, below the bases of the houses. Notice the x-coordinates of the house have been modified under the transformation. The y and z coordinates are fixed. What kind of linear transformation of 3-space is this?

- ○ dilation
- ○ projection
- ○ reflection
- ○ rotation
- ○ shear
- ○ translation

Guess the Transformation 2

In this second transformation graph, the z axis is pointed up, to the left of the houses.



What kind of linear transformation is this second one?

- ○ dilation
- ○ projection
- ○ reflection
- ○ rotation
- ○ shear
- ○ translation

In transformation 2, what vectors go to their negative coordinates, $A\vec{x} = -\vec{x}$?

○vectors on the x-axis

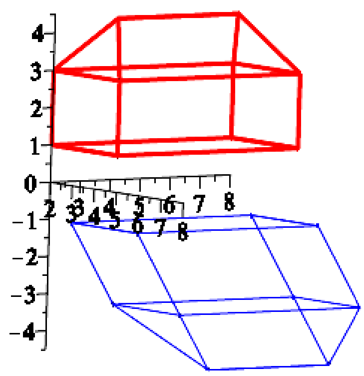
○vectors on the y-axis

○vectors on the z-axis

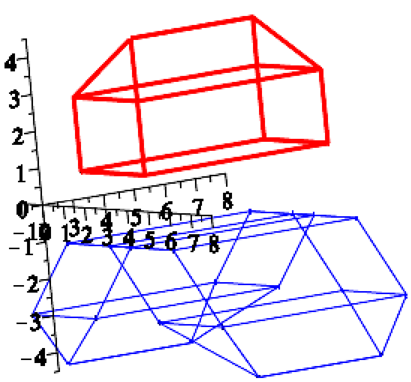
○other

Composition of Transformations

In this visualization I have performed the composition of the two transformations as
Transformation 2 Transformation 1 house



Next I did
Transformation 1 Transformation 2 house
and added it to the same graph, so that the graph shows the original house and outputs of the two compositions: Transformation 1 Transformation 2 house as well as Transformation 2 Transformation 1 house



What does this show?

○matrix multiplication is not commutative

○matrix multiplication is not associative

○other

Check

Question **10**

Not complete

Points out of 4.00

Let *A* be a 2×2 matrix representation of a rotation counterclockwise about the origin.

Use only geometric reasoning of transformations to specify the column space of the rotation matrix, the possible outputs of the linear transformation acting on 2-space? Visualize in your head a rotation of the plane.

- ○ the origin, where we rotate about
- ○ a line in $\mathbb{R}^2$
- ○ all of $\mathbb{R}^2$
- ○ other

Use only geometric reasoning of transformations, what is the null space of the rotation matrix, i.e. what vectors can be sent to the origin under this linear transformation acting on 2-space, i.e. what is smushed to the origin by the linear transformation? Visualize in your head a rotation of the plane---is anything squished to the origin?

- ○ only the origin, where we rotate about
- ○ a line in $\mathbb{R}^2$
- ○ all of $\mathbb{R}^2$
- ○ other

Next, open
https://www.geogebra.org/m/uct4xgv5
and set $a = 0, b = -1, c = 1, d = 0$ to visualize the rotated Mona Lisa and the outputs of all the vectors making up the Mona Lisa image. What is the angle of rotation?

- ○ 0
- ○ $\frac{\pi}{2}$
- ○ π

Let *A* be a 2×2 matrix representation of a projection onto a line in $\mathbb{R}^2$.

Use only geometric reasoning of transformations to specify the column space of the projection matrix, the possible outputs of the linear transformation acting on 2-space?

- ○ the origin
- ○ the line of projection in $\mathbb{R}^2$
- ○ the line perpendicular to the line of projection in $\mathbb{R}^2$
- ○ all of $\mathbb{R}^2$
- ○ other

Revisit
https://www.geogebra.org/m/uct4xgv5
and set $a = 0.5, b = 0.5, c = 0.5, d = 0.6$ --- where d=0.6, not 0.5! Notice that the Mona Lisa image has almost been squished to a line, but not quite. What line?

- ○ x-axis

○y-axis

○$y = x$ line

○$y = -x$ line

Use only geometric reasoning of transformations, what is the null space of a projection matrix, i.e. what vectors can be sent to the origin under a projection acting on 2-space, i.e. what is smushed to the origin by the linear transformation?

○only the origin

○the line of projection in $\mathbb{R}^2$

○the line perpendicular to the line of projection in $\mathbb{R}^2$

○all of $\mathbb{R}^2$

○other

Check

Question **11**

Not complete

Points out of 2.00

For a projection matrix acting on 2-space, notice that vectors on the line of projection are fixed and vectors perpendicular to the line of projection are squished to the origin.

Visualize projection onto the x-axis in your mind and/or open
https://www.geogebra.org/m/uct4xgv5
and set $a = 1, b = 0, c = 0, d = -0.1$ --- where d=-0.1, not 0 and the Mona Lisa image has almost been squished to the x-axis, but instead is seen just below it.

What algebra of the linear transformations corresponds to vectors on the line of projection (the x-axis) in this case) being fixed?

○$A\vec{x} = \vec{x}$

○$A\vec{x} = -\vec{x}$

○$A\vec{x} = \vec{0}$

What algebra of the linear transformations corresponds to vectors perpendicular to the line of projection (the y-axis) in this case) being squished to the origin?

○$A\vec{x} = \vec{x}$

○$A\vec{x} = -\vec{x}$

○$A\vec{x} = \vec{0}$

Check

Question **12**

Not complete

Points out of 2.00

The Hill cipher is a linear transformation of the digitized alphabet via A uncoded message = coded message or equivalently uncoded message = A^{-1}coded message. How does the uncoded message go in?

- ○ as row vectors
- ○ as column vectors

To transform Yoda, why did we need the transpose?

- ○ transpose is the linear transformation that allows us to physically move Yoda from one point in space to another point in space
- ○ Yoda was given as row vectors
- ○ both of the above
- ○ neither

Why is left multiplication used to multiply transformations by an object (i.e. CBAobject, where is is what is first applied) rather than right multiplication?

- ○ With the exception of Yoda, our objects are made of column vectors and so the right matrix multiplication $x \rightarrow \vec{x}A$ doesn't make sense
- ○ matrix multiplication is not commutative
- ○ both of the above
- ○ neither

Check

Question **13**

Not complete

Points out of 1.00

To solidify and prepare for upcoming work, review and contemplate your knowledge and any questions that remain as related to definitions, concepts, computations, and examples from 2.7, including

- 2D and 3D computer graphics as columns of a matrix --- connect the dots!
- effects of 2D and 3D linear transformations from 1.8 and 1.9 and 3D transformations on figures--- algebra and matrix representation and geometry and visualization
- homogeneous coordinates
- composite transformations ABC is read right to left like functions, where C is the first action
- rotate about a point other than the origin (Figure 7)

and consider 6.1, including

- inner product of $\vec{u}$ and $\vec{v}$ and connection to the dot product
- length or norm of a vector
- orthogonal vectors

and 1.8 and 1.9, including

- linear transformation: addition and scalar multiplication
- left multiplication matrix representations
- dilation, projection, reflection, rotation, shear (see Examples 2-5 in 1.8, Examples 2-3 in 1.9, and tables 1-4 in 2.8)
- the algebraic image of the unit axes as a way to find the matrix of the transformation
- range of a linear transformation: the algebraic or geometric images or outputs, e.g. of the unit square as a way to visualize the transformation and understand its effects
- the range, image or output of a sheared sheep

Since the material builds on itself, consider whether there is any material from before this module that you want to brush up on:

1.1

- algebra of linear equations: coefficients and variables
- geometry of linear equations in 2D and 3D: lines and planes
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions
- matrix of a linear system: coefficient matrix, augmented matrix, triangular form
- row equivalent systems
- algorithm for solving a linear system using elementary row operations of replacement, interchange, and scaling

1.2

- matrix of a linear system: row echelon form (Gaussian), reduced row echelon form (Gauss-Jordan)
- pivots: pivot position of a matrix, pivot column of a matrix
- row reduction algorithm we will most commonly use: elimination by forward phase and back substitution to row echelon form
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions with free variables and parametric solutions

1.3

- algebra of vectors: coordinates, addition of vectors, scalar multiplication of vectors, properties like associativity under addition (property ii on p. 29), a linear combination with weights, zero vector, span of a set of vectors=all the linear combinations, is a vector in the span?, vector equation $\longrightarrow$ augmented matrix
- geometry of vectors in 2D and 3D: directed segment, parallelogram for addition, on same line for scalar multiplication of vectors, origin=zero vector, a linear combination geometrically in the plane or 3D, span=all the linear combinations geometrically in the plane or 3D, spaces of subsets of R^n spanned by vectors

1.4

- algebra of matrix vector equation $A\vec{x} = \vec{b}$:
  - multiply a matrix and a column vector by linear combinations of the columns of *A* using weights from $\vec{x}$
  - span of the columns of *A* = set of all linear combinations of the columns of *A*
  - matrix vector equation $\longrightarrow$ vector equation $\longrightarrow$ augmented matrix
  - equations generic vectors $\vec{b}$ must satisfy to be in the span (Example 3)

- dot products of rows of *A* with $\vec{x}$,
- geometry of solutions of matrix vector equation $A\vec{x} = \vec{b}$: spaces of subsets of R^3 spanned by the column vectors of A, geometry of such spaces (Figure 1)
- Theorem 4: relationship of consistency of $A\vec{x} = \vec{b}$ to always being a linear combination to spanning the entire R^m, where *m* is the number of rows, to having a pivot position in every row of *A*.
- identity matrix *I*

1.5

- algebra of homogeneous systems: $A\vec{x} = \vec{0}$
- algebraic solutions of homogenous systems always include the trivial solution=$\vec{0}$. nontrivial solutions, if any exist, are parameterized in parametric vector form using free variables to express those as well as the variables with pivots and then decomposed algebraically to showcase the algebra and geometry giving $t\vec{v}$ or $s\vec{u} + t\vec{v}$ or similar, where each free variable is attached to a vector.
- geometry of solutions of homogeneous systems are geometric spaces through the origin like lines, planes, or hyperplanes
- algebra of nonhomogeneous systens: $A\vec{x} = \vec{b}$
- solutions of nonhomogeneous systems in parametric vector form can be decomposed algebraically to showcase the algebra and geometry like $\vec{p} + t\vec{v}$, vectors ending on the line parallel to $\vec{v}$ or $\vec{p} + s\vec{u} + t\vec{v}$, vectors ending on the plane parallel to the one spanned by $\vec{u}$, $\vec{v}$...
- geometry of solutions of nonhomogeneous systems are geometric spaces translated away from the origin via adding $\vec{p}$

1.7

- linearly independent set of vectors and connection to a homogeneous equation having only the trivial solution
- linearly dependent set of vectors and connection to nontrivial solutions existing and providing a dependance relation
- geometry of linearly independent set of 2 vectors: independent directions in space versus along the same line (Figure 1)
- geometry of linearly independent set of 3 or more vectors: no one vector is in the span of the rest, i.e. they are all needed to span the space versus redundancy in the geometric space they span in the sense that they aren't all needed to generate the same space under linear combinations (Figure 2)
- linearly independent columns of a matrix
- redundancy of $\vec{0}$ in a set of vectors $\{\vec{v}_1 = \vec{0}, \vec{v}_2, \ldots \vec{v}_n\}$ (Theorem 9)

2.1

- matrices: diagonal matrix [and main diagonal], zero matrix
- matrix operations: matrix addition, scalar multiplication of a matrix, matrix multiplication, powers of a matrix, left (or right) multiplication, transpose of a matrix
- matrix multiplication by linear combinations of the columns of A using weights from the corresponding column of B or by the dot products of a row of A with the corresponding column of B.
- algebraic properties that do hold for matrix multiplication: associativity and one-sided distributivity
- algebraic properties that don't hold for matrix multiplication: commutativity

2.2

- matrices: invertible (nonsingular) matrix, noninvertible (singular) matrix, elementary matrix
- determinant and inverse of a 2x2 matrix
- connection between invertibility and unique solutions
- inverse of a product of matrices and inverse of a transpose

2.3

- what makes a matrix invertible for a square matrix (Theorem 8 statements aside from f. and i., which we haven't covered)
- condition number (numerical note on p. 123)

2.8

- subspace properties: closed under addition and scalar multiplication
- spaces associated to a matrix: column space and null space
- basis: linearly independent spanning set
- basis for column space as the pivot columns
- basis for null space as the vectors attached to free variables in parametric solutions of the homogeneous system $A\vec{x} = \vec{0}$

2.9

- dimension of a space

- rank of a matrix (dimension of column space)
- nullity of a matrix (dimension of null space)
- rank nullity theorem (Theorem 14)
- what makes a matrix invertible continued: adding rank and nullity to Theorem 8 when the matrix is square

When you have finished reviewing and reflecting, select one of the following (both receive full credit)

○ I currently have no questions

○ I will continue solidifying and understand that help is available in Dr. Sarah's more extensive feedback that follows below each question after I finish and open back up an entire practice quiz (this is more extensive than the hints that I can access during the open quiz), in Dr. Sarah's glossary/Wiki which is embedded into ASULearn from the linked terms, in Dr. Sarah's office hours and forum, and in Math Lab and Tutoring

Check