You can preview this quiz, but if this were a real attempt, you would be blocked because:

This quiz is not currently available

Question **1**

Not complete

Points out of 4.00

We look at solutions to $A\vec{x} = \vec{0}$ to check that a vector is in the

○null space of *A*

○column space of *A*

We look at whether the vector is in the span of the columns of *A*, i.e. whether the system $A\vec{x} = \vec{b}$ is consistent, where the vector is $\vec{b}$ to check that a vector is in the

○null space of *A*

○column space of *A*

A subspace is closed under

○addition and scalar multiplication (it's linear!)

○dot products

○matrix multiplication

A basis for a subspace is a set of vectors that is

○homogeneous

○closed under addition and scalar multiplication (it's linear)

○linear independent and spans the subspace

Check

Question **2**

Not complete

Points out of 1.00

Which of the following is not a subspace of $\mathbb{R}^n$?

Choose all that apply

- ☐ the span of a set of vectors in $\mathbb{R}^n$
- ☐ The set of all linear combinations of a set of vectors in $\mathbb{R}^n$
- ☐ $\vec{0}$
- ☐ A line l through the origin, like the line $t\begin{bmatrix}1\\2\\3\end{bmatrix}$
- ☐ A line not through the origin, like the line parallel to $\begin{bmatrix}1\\2\\3\end{bmatrix}$ and through the tip of $\begin{bmatrix}4\\5\\6\end{bmatrix}$, i.e. the line $t\begin{bmatrix}1\\2\\3\end{bmatrix}+\begin{bmatrix}4\\5\\6\end{bmatrix}$
- ☐ $\mathbb{R}^n$

Check

Question **3**

Not complete

Points out of 2.00

Open

https://www.geogebra.org/m/qxgg7vrm

Move the sliders x_1, x_2 to visualize the column space of a matrix A, i.e. the span of its columns, like we did back in chapter 1 when we looked at the linear combinations of columns of a matrix (but now we have a new name for it!). In this visualization, what is the column space?

- ○ infinite line
- ○ infinite plane
- ○ other

Generally, the nullspace of A, which is the solutions to $A\vec{x}=\vec{0}$, like we solved for back in chapter 1 (but now we have a new name!) is inside a different space (like it could be 4-space or 2-space when the column space is in 3-space, or vice a versa). However, the nullspace of the transpose A^T is inside the same space! And this has cool real-life applications too, like to least squares and linear regression.

Next, move the slider x_3 in

https://www.geogebra.org/m/qxgg7vrm

for the nullspace of A^T. What is the nullspace of A^T?

- ○ only the origin
- ○ infinite line
- ○ infinite plane
- ○ other

Check

Question **4**

Not complete

Points out of 32.00

To solve for the nullspace of $\begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ augment with $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. What is the corresponding augmented matrix?

[ ]	[ ]	[ ]	[ ]
[ ]	[ ]	[ ]	[ ]
[ ]	[ ]	[ ]	[ ]

What is the first step of strict Gaussian (use replacement but not scaling) and for the purpose of ASULearn, don't swap rows either, although that is allowed in strict Gaussian.

$r_3' =$

[ ]

$r_1 + r_3$

In your notes, write the matrix that you obtain after this one replacement. What is the next step in strict Gaussian?

$r_3' =$

[ ]

$r_2 + r_3$

What is the augmented matrix after applying these two replacements, i.e. the row echelon form?

[ ]	[ ]	[ ]	[ ]
[ ]	[ ]	[ ]	[ ]
[ ]	[ ]	[ ]	[ ]

Which variables have pivots?

- ○ only x_1
- ○ only x_2
- ○ only x_3
- ○ x_1 and x_2
- ○ x_1 and x_3
- ○ x_2 and x_3
- ○ x_1, x_2, and x_3

Notice that the augmented matrix is consistent since there is no row of [0 0 0 nonzero] in row echelon form. Any variables without pivots in a consistent system in row echelon form are free. Which variables are free?

○ x_1

○ only x_2

○ only x_3

○ x_1 and x_2

○ x_1 and x_3

○ x_2 and x_3

○ x_1, x_2, and x_3

Keep the free variables as free parameters. We can set them as time parameters, for instance, or keep them as they are listed. So for example, if there is 1 variable missing pivots in the reduced matrix, it can get a variable like *t* and if 2 were missing we could have an *s* and *t*. Then solve for any variables with pivots in terms of the parameters using the equations corresponding to any nonzero rows in row echelon form.

What is x_2?

○ it stays as x_2 or equivalently a parameter t since it is free

○ x_1

○ x_3

○ $-2x_3$

○ other

What is x_1?

○ it stays as x_1 or equivalently a parameter t since it is free

○ x_2

○ x_3

○ $-x_3$

○ other

Next write the solutions in vector form as a column vector. The free variables stay free while the pivot variables are written in terms of the parameters. Using this method of parameterization, which of the following represents the solutions at this step?

○ $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

○ $\begin{bmatrix} -x_3 \\ 2x_3 \\ 3x_3 \end{bmatrix}$

○ $\begin{bmatrix} -x_3 \\ -2x_3 \\ x_3 \end{bmatrix}$

○ other

The last step of parameterization for the entire nullspace is to factor out any free variables. Which of the following describes the nullspace and in geometrically and in parametric vector form?

- ◯ no solutions as there are no concurrent intersections
- ◯ a line of solutions $x_3 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
- ◯ a line of solutions $x_3 \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$
- ◯ a plane of solutions $x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$
- ◯ an infinite volume of solutions $x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$
- ◯ other

Check

Question **5**

Not complete

Points out of 11.00

Look at $A = \begin{bmatrix} -1 & 2 & -1 \\ 1 & -2 & 2 \\ 2 & -4 & 2 \end{bmatrix}$. Reduce *A* to row echelon form using strict Gaussian (yes replacement, but no scaling) and don't swap rows either. What is the row echelon form?

-1	2	-1
☐	☐	☐
☐	☐	☐

To solve for the column space and null space by-hand, which columns of the original matrix are in a basis for the column space from our usual method of taking the pivot columns?

- ○ only column 1
- ○ only column 2
- ○ only column 3
- ○ column 1 and column 2
- ○ column 1 and column 3
- ○ column 2 and column 3
- ○ all 3 columns

The number of vectors in a basis determines what kind of object we form when take the span, i.e. all linear combinations of the basis. What kind of geometric object is the column space?

- ○ a point in $\mathbb{R}^3$
- ○ 2 points in $\mathbb{R}^3$
- ○ a line in $\mathbb{R}^3$
- ○ a plane in $\mathbb{R}^3$
- ○ all of $\mathbb{R}^3$
- ○ other

To solve for the nullspace, we solve $A\vec{x} = \vec{0}$ by parameterizing any free variables and solving the system. So for example, if there is 1 variable missing pivots in the reduced matrix, it gets a variable like *t* and if 2 were missing we would have an *s* and *t*. Which variables are free, without pivots?

- ○ only x_1
- ○ only x_2
- ○ only x_3
- ○ x_1 and x_2
- ○ x_1 and x_3
- ○ x_2 and x_3
- ○ x_1, x_2, and x_3

Then we use the rows with pivots to solve for the other variables in terms of the parameters. Next we write out the solutions as a column vector and factor out any free variables. The basis is what spans the nullspace and we get that by recognizing which vectors span the entire nullspace (i.e the individual vectors). For instance, if the entire nullspace is s(vector1) + t(vector2) then {vector 1, vector2} would be a basis. Write a basis for the nullspace. How many vectors does it have?

- ○0
- ○1
- ○2
- ○3

The number of vectors in a basis determines what kind of object we form when take the span, i.e. all linear combinations of the basis. What kind of geometric object is the nullspace?

- ○a point in $\mathbb{R}^3$
- ○2 points in $\mathbb{R}^3$
- ○a line in $\mathbb{R}^3$
- ○a plane in $\mathbb{R}^3$
- ○all of $\mathbb{R}^3$
- ○other

Check

Question **6**

Not complete

Points out of 2.00

This is adapted from a question you had in 1.5.

Look at $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$. Reduce *A* to row echelon form using strict Gaussian (yes replacement, but no scaling) and don't swap rows either. What is the row echelon form?

☐	☐	☐
☐	☐	☐

To solve for the nullspace, we solve $A\vec{x} = \vec{0}$. Which variables are free?

- ○ only x_1
- ○ only x_2
- ○ only x_3
- ○ x_1 and x_2
- ○ x_1 and x_3
- ○ x_2 and x_3

Solve the system $A\vec{x} = \vec{0}$ by parameterizing the solutions. What is the nullspace of A?

- ○ no solutions as there are no concurrent intersections
- ○ a line of solutions $x_3 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
- ○ a plane of solutions $x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$
- ○ other

The basis is the individual vectors that span the nullspace. Which vectors are in a basis for the nullspace?

- ○ there is no basis
- ○ $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
- ○ $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$
- ○ other

To solve for the column space, we look at the span of the column vectors. Is it true that the column space of A equals $t \begin{bmatrix} 1 \\ 2 \end{bmatrix}$?

- ○ yes
- ○ no

The pivot columns of A form a basis for the column space. Is $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$ a basis for the column space?

- ◯ yes
- ◯ no

Check

Question **7**

Not complete

Points out of 5.00

Let $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$

which reduces to $\begin{bmatrix} 1 & -2 & 0 & -1 & 3 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

The pivot columns form a basis for the column space, the span of the columns. What is a basis for the column space?

○ $x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

○ $\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} \}$

○ $\{ \begin{bmatrix} 6 \\ -2 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 8 \end{bmatrix}, \begin{bmatrix} -7 \\ -1 \\ -4 \end{bmatrix} \}$

○ $\{ \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -0 \end{bmatrix} \}$

○ $\{ \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix} \}$

○ $\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \}$

○ more than one above

○ other

What is the geometry of the entire column space, the span of the columns?

○ an infinite volume

○ a plane in $\mathbb{R}^2$

○ a plane in $\mathbb{R}^3$

○ two points

Notice that $[A\vec{0}]$ reduces to $\begin{bmatrix} 1 & -2 & 0 & -1 & 3 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Is x_3 free in $[A\vec{0}]$ and why or why not?

○yes, x_3 is free as it is missing a pivot

○no, x_3 is not free as it has a pivot

○other

Take the variables that are free, those missing a pivot, and keep them as the parameters. Then use the other rows to solve for the ones that had pivots. For example, row 1 has a pivot for x_1 and we can solve for it using $x_1 - 2x_2 - x_4 + 3x_5 = 0$, just like we did back in 1.5. Then write the solutions in parameterized vector form. What is a basis for null space?

○$x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

○$\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} \}$

○$\{ \begin{bmatrix} 6 \\ -2 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 8 \end{bmatrix}, \begin{bmatrix} -7 \\ -1 \\ -4 \end{bmatrix} \}$

○$\{ \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix} \}$

○other

What is the geometry of the entire null space?

○an infinite volume generated by the 3 free variables

○a plane in $\mathbb{R}^2$

○a plane in $\mathbb{R}^3$

○two points

Check

Question **8**

Not complete

Points out of 4.00

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

This is already reduced so write a basis for the column space in your notes via the pivot columns. Next, what is the entire column space (the span of the pivot columns)?

- ○ the origin
- ○ the *x*-axis
- ○ the *y*-axis
- ○ all of $\mathbb{R}^2$
- ○ other

To find the null space, look for free variables in $A\vec{x} = \vec{0}$ and parameterize the solutions to this homogeneous equation. Then solve for variables with pivots using the equations corresponding to those rows. Next write out the solutions in parameterized vector form. What is the null space?

- ○ the origin
- ○ the *x*-axis
- ○ the *y*-axis
- ○ all of $\mathbb{R}^2$
- ○ other

We'll see why this is called a projection matrix later on!

Check

Question **9**

Not complete

Points out of 4.00

Provide integers *p* and *q* such that the nullspace of *A* is a subspace of $\mathbb{R}^p$ and the column space of *A* is a subspace of $\mathbb{R}^q$, where

$$A = \begin{bmatrix} 3 & 2 & 1 & -5 \\ -9 & -4 & 1 & 7 \\ 9 & 2 & -5 & 1 \end{bmatrix}.$$

What is *p* for the nullspace of *A* as a subspace of $\mathbb{R}^p$?

[ ]

What is *q* for the column space of *A* as a subspace of $\mathbb{R}^q$

[ ]

Reduce *A* to row echelon form using strict Gaussian (yes replacement, but no scaling) and don't swap rows either. What is the row echelon form?

3	2	1	-5
[ ]	[ ]	[ ]	[ ]
[ ]	[ ]	[ ]	[ ]

Next, solve for the column space and null space and bases for them by-hand in your notes.

Which of the following are in the nullspace?

- ○ $\begin{bmatrix} 3 \\ -9 \\ 9 \end{bmatrix}$
- ○ $\begin{bmatrix} 1 \\ 1 \\ -5 \end{bmatrix}$
- ○ $\begin{bmatrix} 3 \\ 2 \\ 1 \\ -5 \end{bmatrix}$
- ○ $\begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$
- ○ more than one of these

How many vectors are in a basis for the nullspace?

[ ]

Which of the following are in the column space?

- ○ $\begin{bmatrix} 3 \\ -9 \\ 9 \end{bmatrix}$
- ○ $\begin{bmatrix} 1 \\ 1 \\ -5 \end{bmatrix}$
- ○ $\begin{bmatrix} 3 \\ 2 \\ 1 \\ -5 \end{bmatrix}$
- ○ $\begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$
- ○ more than one of these

Which of the following are in the basis for the column space using the pivot columns?

- ○ $\begin{bmatrix} 3 \\ -9 \\ 9 \end{bmatrix}$
- ○ $\begin{bmatrix} 1 \\ 1 \\ -5 \end{bmatrix}$
- ○ $\begin{bmatrix} 3 \\ 2 \\ 1 \\ -5 \end{bmatrix}$
- ○ $\begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$
- ○ more than one of these

How many vectors are in a basis for the column space?

[ ]

Check

Question **10**

Not complete

Points out of 1.00

Determine if the set of vectors is a basis for $\mathbb{R}^3$: $\begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$

Examine the following arguments for their validity:

Argument 1: Look at $A = \begin{bmatrix} 0 & 5 & 6 \\ 0 & 0 & 3 \\ -2 & 4 & 2 \end{bmatrix}$, which is the square matrix with the vectors as columns. To put *A* in row echelon form, swap the rows of *A:* first swap row 1 and 3 and then swap row 2 with new row 3. Now *A* is row equivalent to a matrix in row echelon form . We see full row pivots so the system $A\vec{x} = \vec{b}$ is never inconsistent and the set of column vectors of *A* span $\mathbb{R}^3$. In addition, in this example we also have full column pivots so $A\vec{x} = \vec{0}$ has only the trivial solution and the set of column vectors is linearly independent. Thus the set of vectors satisfy the definitions of span the entire space and linearly independent, and they form a basis

Argument 2: The square matrix $A = \begin{bmatrix} 0 & 5 & 6 \\ 0 & 0 & 3 \\ -2 & 4 & 2 \end{bmatrix}$ is not invertible so by the negation of what makes a matrix invertible theorem, the columns of A do not span $\mathbb{R}^3$ and are not linearly independent. So the vectors do not form a basis.

Which, if any, arguments are valid here?

- ○ argument 1
- ○ argument 2
- ○ neither

Check

Question **11**

Not complete

Points out of 1.00

Is the statement "The columns of an invertible *nxn* matrix form a basis for $\mathbb{R}^n$" true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

- ○ True and I found a phrase and page number from the text
- ○ False and I can provide a counterexample

Check

Question **12**

Not complete

Points out of 1.00

Is the statement "The set of all solutions of a system of *m* homogeneous equations in *n* unknowns is a subspace of $\mathbb{R}^m$. " true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

- ○ True and I found a phrase and page number from the text
- ○ False and I can provide a counterexample

Check

Question **13**

Not complete

Points out of 1.00

To solidify and prepare for upcoming work, review and contemplate your knowledge and any questions that remain as related to definitions, concepts, computations, and examples from 2.8, including

- subspace properties: closed under addition and scalar multiplication
- spaces associated to a matrix: column space and null space
- basis: linearly independent spanning set
- basis for column space as the pivot columns
- basis for null space as the vectors attached to free variables in parametric solutions of the homogeneous system $A\vec{x} = \vec{0}$

and consider 2.3, including

- what makes a matrix invertible for a square matrix (Theorem 8 statements aside from f. and i., which we haven't covered)
- condition number (numerical note on p. 123)

and 2.2, including

- matrices: invertible (nonsingular) matrix, noninvertible (singular) matrix, elementary matrix
- determinant and inverse of a 2x2 matrix
- connection between invertibility and unique solutions
- inverse of a product of matrices and inverse of a transpose

and 2.1, including

- matrices: diagonal matrix [and main diagonal], zero matrix
- matrix operations: matrix addition, scalar multiplication of a matrix, matrix multiplication, powers of a matrix, left (or right) multiplication, transpose of a matrix
- matrix multiplication by linear combinations of the columns of A using weights from the corresponding column of B or by the dot products of a row of A with the corresponding column of B.
- algebraic properties that do hold for matrix multiplication: associativity and one-sided distributivity
- algebraic properties that don't hold for matrix multiplication: commutativity

Since the material builds on itself, consider whether there is any material from before this module that you want to brush up on:

1.1

- algebra of linear equations: coefficients and variables
- geometry of linear equations in 2D and 3D: lines and planes
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions
- matrix of a linear system: coefficient matrix, augmented matrix, triangular form
- row equivalent systems
- algorithm for solving a linear system using elementary row operations of replacement, interchange, and scaling

1.2

- matrix of a linear system: row echelon form (Gaussian), reduced row echelon form (Gauss-Jordan)
- pivots: pivot position of a matrix, pivot column of a matrix
- row reduction algorithm we will most commonly use: elimination by forward phase and back substitution to row echelon form
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions with free variables and parametric solutions

1.3

- algebra of vectors: coordinates, addition of vectors, scalar multiplication of vectors, properties like associativity under addition (property ii on p. 29), a linear combination with weights, zero vector, span of a set of vectors=all the linear combinations, is a vector in the span?, vector equation $\longrightarrow$ augmented matrix
- geometry of vectors in 2D and 3D: directed segment, parallelogram for addition, on same line for scalar multiplication of vectors, origin=zero vector, a linear combination geometrically in the plane or 3D, span=all the linear combinations geometrically in the plane or 3D, spaces of subsets of R^n spanned by vectors

1.4

- algebra of matrix vector equation $A\vec{x} = \vec{b}$:
  - multiply a matrix and a column vector by linear combinations of the columns of *A* using weights from $\vec{x}$
  - span of the columns of *A* = set of all linear combinations of the columns of *A*
  - matrix vector equation $\longrightarrow$ vector equation $\longrightarrow$ augmented matrix
  - equations generic vectors $\vec{b}$ must satisfy to be in the span (Example 3)
  - dot products of rows of *A* with $\vec{x}$,
- geometry of solutions of matrix vector equation $A\vec{x} = \vec{b}$: spaces of subsets of R^3 spanned by the column vectors of A, geometry of such spaces (Figure 1)
- Theorem 4: relationship of consistency of $A\vec{x} = \vec{b}$ to always being a linear combination to spanning the entire R^m, where *m* is the number of rows, to having a pivot position in every row of *A*.
- identity matrix *I*

1.5

- algebra of homogeneous systems: $A\vec{x} = \vec{0}$
- algebraic solutions of homogenous systems always include the trivial solution=$\vec{0}$. nontrivial solutions, if any exist, are parameterized in parametric vector form using free variables to express those as well as the variables with pivots and then decomposed algebraically to showcase the algebra and geometry giving $t\vec{v}$ or $s\vec{u} + t\vec{v}$ or similar, where each free variable is attached to a vector.
- geometry of solutions of homogeneous systems are geometric spaces through the origin like lines, planes, or hyperplanes
- algebra of nonhomogeneous systens: $A\vec{x} = \vec{b}$
- solutions of nonhomogeneous systems in parametric vector form can be decomposed algebraically to showcase the algebra and geometry like $\vec{p} + t\vec{v}$, vectors ending on the line parallel to $\vec{v}$ or $\vec{p} + s\vec{u} + t\vec{v}$, vectors ending on the plane parallel to the one spanned by $\vec{u}$, $\vec{v}$...
- geometry of solutions of nonhomogeneous systems are geometric spaces translated away from the origin via adding $\vec{p}$

1.7

- linearly independent set of vectors and connection to a homogeneous equation having only the trivial solution
- linearly dependent set of vectors and connection to nontrivial solutions existing and providing a dependance relation
- geometry of linearly independent set of 2 vectors: independent directions in space versus along the same line (Figure 1)
- geometry of linearly independent set of 3 or more vectors: no one vector is in the span of the rest, i.e. they are all needed to span the space versus redundancy in the geometric space they span in the sense that they aren't all needed to generate the same space under linear combinations (Figure 2)
- linearly independent columns of a matrix
- redundancy of $\vec{0}$ in a set of vectors $\{\vec{v}_1 = \vec{0}, \vec{v}_2, \ldots \vec{v}_n\}$ (Theorem 9)

When you have finished reviewing and reflecting, select one of the following (both receive full credit)

○ I currently have no questions

○ I will continue solidifying and understand that help is available in Dr. Sarah's more extensive feedback that follows below each question after I finish and open back up an entire practice quiz (this is more extensive than the hints that I can access during the open quiz), in Dr. Sarah's glossary/Wiki which is embedded into ASULearn from the linked terms, in Dr. Sarah's office hours and forum, and in Math Lab and Tutoring

Check