Question **1**

Not complete

Points out of 16.00

Look at $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \end{bmatrix}$. Reduce *A* to row echelon form using strict Gaussian (yes replacement, but no scaling) and don't swap rows either. What is the row echelon form?

☐	☐	☐
☐	☐	☐

To solve for the nullspace, we solve $A\vec{x} = \vec{0}$. Which variables are free?

- ○ only x_1
- ○ only x_2
- ○ only x_3
- ○ x_1 and x_2
- ○ x_1 and x_3
- ○ x_2 and x_3

Our usual method for the nullspace is to parameterize and solve $A\vec{x} = \vec{0}$ in terms of the free variables and then factor. The individual vectors in this parameterization form a basis for the nullspace and they generate the entire nullspace via their linear combinations. Write the basis for the nullspace of A from our usual method of solving for it.

☐
☐
☐

What is the geometry of the null space?

- ○ $\vec{0}$
- ○ a line in $\mathbb{R}^2$
- ○ all of $\mathbb{R}^2$
- ○ a line in $\mathbb{R}^3$
- ○ a plane in $\mathbb{R}^3$
- ○ all of $\mathbb{R}^3$

To solve for the column space, we look at the span of the column vectors and the pivot columns of *A* form a basis for the column space. Is $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$ a basis for the column space?

- ○ yes
- ○ no

What is the geometry of the column space?

○$\vec{0}$

○a line in $\mathbb{R}^2$

○all of $\mathbb{R}^2$

○a line in $\mathbb{R}^3$

○a plane in $\mathbb{R}^3$

○all of $\mathbb{R}^3$

What is the rank?

What is the nullity?

What is rank + nullity?

Check

Question **2**

Not complete

Points out of 1.00

What is the difference between a basis and the subspace?

○they are the same

○A basis is set of vectors that are linear independent span the subspace. We take linear combinations of the basis to generate the subspace.

○A subspace is a set that is closed under addition and scalar multiplication. We take linear combinations of the subspace to get the basis

Check

Question **3**

Not complete

Points out of 4.00

Suppose that *A* is a 5×6 matrix with exactly 4 linearly independent columns.

A has rank

[ ]

and nullity

[ ]

.

What does 5 stand for in this 5×6 matrix?

- ○ the dimension of the column space
- ○ the dimension of the Euclidean space that the column space sits inside of, i.e. the number of coordinates each column vector has
- ○ the dimension of the null space
- ○ the dimension of the Euclidean space that the null space sits inside of, i.e. the number of coordinates each vector in the nullspace has

What does 6 stand for in this 5×6 matrix?

- ○ the dimension of the column space
- ○ the dimension of the Euclidean space that the column space sits inside of, i.e. the number of coordinates each column vector has
- ○ the dimension of the null space
- ○ the dimension of the Euclidean space that the null space sits inside of, i.e. the number of coordinates each vector in the nullspace has

Where does a basis for the nullspace come from?

- ○ the pivot columns
- ○ the rows that have pivots
- ○ the vectors that span the parameterized vector solutions of $A\vec{x} = \vec{0}$

Check

Question **4**

Not complete

Points out of 3.00

Open
https://www.geogebra.org/m/qxgg7vrm
Move the sliders x_1, x_2 to visualize the column space of a matrix A, i.e. the span of its columns. What is the rank?

- ○0
- ○1
- ○2
- ○3

Generally, the nullspace of A is inside a different space (like it could be 4-space or 2-space when the column space is in 3-space, or vice a versa). However, the nullspace of the transpose A^T is inside the same space! And this has cool real-life applications too, like to least squares and linear regression. So, move the slider x_3 for the nullspace of A^T. What is the nullity?

- ○0
- ○1
- ○2
- ○3

What is the rank + nullity?

Check

Question **5**

Not complete

Points out of 1.00

Least squares analysis is very useful in many real-life applications where we want a best fit line (it's linear!). However, typically, no line exactly goes through all of the given points. We want to find the line that fits as closely as possible to all of the points (we minimize the squared vertical distance between the points and the line using the Pythagorean theorem).

Typically the matrix we are looking at isn't square, so we can't take its inverse, but we can take the inverse of (A^TA)! Then the key to finding the y-intercept and slope $\begin{bmatrix} b \\ m \end{bmatrix}$ of the best fit line makes use of matrix algebra $(A^TA)^{-1}A^T\vec{y}$ as well as the null space of A^T and the column space of A. We have been computing the column space of a matrix in 2.8 via reducing to row echelon form and identifying the pivot columns as a basis. The entire column space is the span of the basis vectors. In 2.8 we also computed the null space of a matrix via augmenting with the 0 vector, reducing, and parameterizing solutions like we did back in 1.5.

In least squares, we still look at the column space of a matrix, but now it is the null space of the transpose of the matrix, not the original.

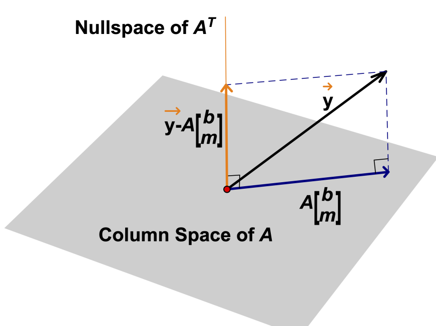
For example, say $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$. To find the null space of A^T, notice that $A^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ and we want to solve for solutions of $A^T\vec{x} = \vec{0}$ and write a basis. The augmented matrix is $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 \end{bmatrix}$ which reduces to $\xrightarrow{r_2' = -r_1 + r_2} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$

Here x_3 is free so set it to t.
Use row 2 to solve for $x_2 = -2t$.
Use row 1 to solve for $x_1 = -x_2 - x_3 = -(-2t) - t = t$.

Thus the nullspace is $t\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ and $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}$ is a basis for that line.

What is the nullity of A^T?

- ○0
- ○1
- ○2
- ○∞
- ○other



We can see that $A\begin{bmatrix} b \\ m \end{bmatrix}$ is the vector in the column space of A that is closest to $\vec{y}$, the projection of $\vec{y}$ onto the plane. In addition, $\vec{y} - A\begin{bmatrix} b \\ m \end{bmatrix}$ is in the null space of A^T, which gives us the way to compute best fit lines! Lots of course topics connect: column space, inverse, matrix algebra, matrix multiplication, nullspace, solving linear equations, and transpose.

Check

Question **6**

Not complete

Points out of 5.00

Look at the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

What is the column space?

- ○ $\vec{0}$
- ○ a line through the origin
- ○ $\mathbb{R}^2$

What is the rank?

[ ]

What is the null space?

- ○ $\vec{0}$
- ○ a line through the origin
- ○ $\mathbb{R}^2$

What is rank + nullity equal to?

[ ]

What is the nullity?

[ ]

Check

Question **7**

Not complete

Points out of 4.00

Look at $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$. In your notes, solve for the nullspace and the column space by hand.

What is the rank?

[ ]

What is the nullity?

[ ]

What is rank + nullity?

[ ]

Open
https://www.geogebra.org/m/n4xwnuuk
If the sliders aren't already both playing, start them. After the two free sliders have had the chance to go around, turn the graph to look for a "head on" view of a line or plane the vectors are in, if that is possible. What does the visualization show?

○ it is a visualization of the column space

○ it is a visualization of the nullspace

○ other

Check

Question **8**

Not complete

Points out of 3.00

Suppose S is a linearly independent set in $\mathbb{R}^3$. Drag the tiles to fill in the blanks.

- S with l.i. vectors cannot contain [ ] 3 vectors in $\mathbb{R}^3$
- If S with l.i. vectors contains [ ] 3 vectors then S is a basis for all of $\mathbb{R}^3$.
- If S with l.i. vectors contains [ ] 3 vectors then S spans a proper subspace of $\mathbb{R}^3$, i.e. the origin, a line through the origin, or a plane through the origin, but not all of $\mathbb{R}^3$.

more than | exactly | less than

Check

Question **9**

Not complete

Points out of 2.00

Suppose A is an $n \times n$ matrix and $\vec{b}$ is a vector in $\mathbb{R}^n$ such that $A\vec{x} = \vec{b}$ has no solution (i.e. is inconsistent). Which of the following are true?

Select one or more:

- [ ] A is invertible
- [ ] The column space of A is equal to $\mathbb{R}^n$
- [ ] The null space of A is at least a line through the origin.
- [ ] The null space of A has nontrivial solutions of $A\vec{x} = \vec{0}$ in it

Check

Question **10**

Not complete

Points out of 1.00

To solidify and prepare for upcoming work, review and contemplate your knowledge and any questions that remain as related to definitions, concepts, computations, and examples from 2.9, including

- dimension of a space
- rank of a matrix (dimension of column space)
- nullity of a matrix (dimension of null space)
- rank nullity theorem (Theorem 14)
- what makes a matrix invertible continued: adding rank and nullity to Theorem 8 when the matrix is square

and consider 2.8, including

- subspace properties: closed under addition and scalar multiplication
- spaces associated to a matrix: column space and null space
- basis: linearly independent spanning set
- basis for column space as the pivot columns
- basis for null space as the vectors attached to free variables in parametric solutions of the homogeneous system $A\vec{x} = \vec{0}$

and 2.3, including

- what makes a matrix invertible for a square matrix (Theorem 8 statements aside from f. and i., which we haven't covered)
- condition number (numerical note on p. 123)

and 2.2, including

- matrices: invertible (nonsingular) matrix, noninvertible (singular) matrix, elementary matrix
- determinant and inverse of a 2x2 matrix
- connection between invertibility and unique solutions
- inverse of a product of matrices and inverse of a transpose

and 2.1, including

- matrices: diagonal matrix [and main diagonal], zero matrix
- matrix operations: matrix addition, scalar multiplication of a matrix, matrix multiplication, powers of a matrix, left (or right) multiplication, transpose of a matrix
- matrix multiplication by linear combinations of the columns of A using weights from the corresponding column of B or by the dot products of a row of A with the corresponding column of B.
- algebraic properties that do hold for matrix multiplication: associativity and one-sided distributivity
- algebraic properties that don't hold for matrix multiplication: commutativity

Since the material builds on itself, consider whether there is any material from before this module that you want to brush up on:

1.1

- algebra of linear equations: coefficients and variables
- geometry of linear equations in 2D and 3D: lines and planes
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions
- matrix of a linear system: coefficient matrix, augmented matrix, triangular form
- row equivalent systems
- algorithm for solving a linear system using elementary row operations of replacement, interchange, and scaling

1.2

- matrix of a linear system: row echelon form (Gaussian), reduced row echelon form (Gauss-Jordan)
- pivots: pivot position of a matrix, pivot column of a matrix
- row reduction algorithm we will most commonly use: elimination by forward phase and back substitution to row echelon form
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions with free variables and parametric solutions

1.3

- algebra of vectors: coordinates, addition of vectors, scalar multiplication of vectors, properties like associativity under addition (property ii on p. 29), a linear combination with weights, zero vector, span of a set of vectors=all the linear combinations, is a vector in the span?, vector equation $\longrightarrow$ augmented matrix
- geometry of vectors in 2D and 3D: directed segment, parallelogram for addition, on same line for scalar multiplication of vectors, origin=zero vector, a linear combination geometrically in the plane or 3D, span=all the linear combinations geometrically in the plane or 3D, spaces of subsets of R^n spanned by vectors

1.4

- algebra of matrix vector equation $A\vec{x} = \vec{b}$:
  - multiply a matrix and a column vector by linear combinations of the columns of *A* using weights from $\vec{x}$
  - span of the columns of *A* = set of all linear combinations of the columns of *A*
  - matrix vector equation $\longrightarrow$ vector equation $\longrightarrow$ augmented matrix
  - equations generic vectors $\vec{b}$ must satisfy to be in the span (Example 3)
  - dot products of rows of *A* with $\vec{x}$,
- geometry of solutions of matrix vector equation $A\vec{x} = \vec{b}$: spaces of subsets of R^3 spanned by the column vectors of A, geometry of such spaces (Figure 1)
- Theorem 4: relationship of consistency of $A\vec{x} = \vec{b}$ to always being a linear combination to spanning the entire R^m, where *m* is the number of rows, to having a pivot position in every row of *A*.
- identity matrix *I*

1.5

- algebra of homogeneous systems: $A\vec{x} = \vec{0}$
- algebraic solutions of homogenous systems always include the trivial solution=$\vec{0}$. nontrivial solutions, if any exist, are parameterized in parametric vector form using free variables to express those as well as the variables with pivots and then decomposed algebraically to showcase the algebra and geometry giving $t\vec{v}$ or $s\vec{u} + t\vec{v}$ or similar, where each free variable is attached to a vector.
- geometry of solutions of homogeneous systems are geometric spaces through the origin like lines, planes, or hyperplanes
- algebra of nonhomogeneous systens: $A\vec{x} = \vec{b}$
- solutions of nonhomogeneous systems in parametric vector form can be decomposed algebraically to showcase the algebra and geometry like $\vec{p} + t\vec{v}$, vectors ending on the line parallel to $\vec{v}$ or $\vec{p} + s\vec{u} + t\vec{v}$, vectors ending on the plane parallel to the one spanned by $\vec{u}, \vec{v}$...
- geometry of solutions of nonhomogeneous systems are geometric spaces translated away from the origin via adding $\vec{p}$

1.7

- linearly independent set of vectors and connection to a homogeneous equation having only the trivial solution
- linearly dependent set of vectors and connection to nontrivial solutions existing and providing a dependance relation
- geometry of linearly independent set of 2 vectors: independent directions in space versus along the same line (Figure 1)
- geometry of linearly independent set of 3 or more vectors: no one vector is in the span of the rest, i.e. they are all needed to span the space versus redundancy in the geometric space they span in the sense that they aren't all needed to generate the same space under linear combinations (Figure 2)
- linearly independent columns of a matrix
- redundancy of $\vec{0}$ in a set of vectors $\{\vec{v}_1 = \vec{0}, \vec{v}_2, \ldots \vec{v}_n\}$ (Theorem 9)

When you have finished reviewing and reflecting, select one of the following (both receive full credit)

○ I currently have no questions

○ I will continue solidifying and understand that help is available in Dr. Sarah's more extensive feedback that follows below each question after I finish and open back up an entire practice quiz (this is more extensive than the hints that I can access during the open quiz), in Dr. Sarah's glossary/Wiki which is embedded into ASULearn from the linked terms, in Dr. Sarah's office hours and forum, and in Math Lab and Tutoring

Check