Question 1

Not complete

Points out of 17.00

Use the diagonal method to compute the determinant of $\begin{bmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{bmatrix}$.

How many diagonals are there in the determinant in this case?

[ ]

What is the determinant using the diagonal method? Simplify your answer and don't put in an extra characters or spaces.

[ ]

The diagonal methods do not generalize in any reasonable way to 4x4 or larger matrices - there we must use the Laplace expansion method. To gain some practice with the Laplace expansion, expand along the first column and fill in the blanks:

The first term in the Laplace expansion along the first column is:

[ ] $(-1)^{1+1}$ times the determinant of: [ ] [ ] / [ ] [ ]

The second term in the Laplace expansion along the first column is:

[ ] $(-1)^{2+1}$ times the determinant of: [ ] [ ] / [ ] [ ]

The third term in the Laplace expansion along the first column is:

[ ] $(-1)^{3+1}$ times the determinant of: [ ] [ ] / [ ] [ ]

Check

Question **2**

Not complete

Points out of 3.00

Explore the impact of an elementary row operation on determinant as follows:

What is the determinant of $A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$?

Simplify your answer and don't put in an extra characters or spaces.

[ ]

Now we modify A via an elementary row operation and turn it into $\begin{bmatrix} 3 & 4 \\ 5+3k & 6+4k \end{bmatrix}$

What row operation turns A into this new matrix?

replacement via $r_2' =$

[ ]

$r_1 + r_2$

What is the impact of this elementary row operation on the determinant?

- ○ multiplies it by k
- ○ preserves it
- ○ other

Check

Question **3**

Not complete

Points out of 2.00

Compute the determinant of the elementary matrix $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & k & 1 \end{bmatrix}$

determinant *E*=

[ ]

What row operation on *B*, a 3x3 matrix, does this matrix represent for the matrix multiplication *EB*?

- ○ $r_2' = kr_3 + r_2$
- ○ $r_3' = kr_1 + r_3$
- ○ $r_3' = kr_2 + r_3$
- ○ other

Check

Question **4**

Not complete

Points out of 1.00

Is the statement "The (*i*,*j*)-cofactor of a matrix *A* is the matrix A_{ij} obtained by deleting from *A* its *i*th row and *j*th column" true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

- ○ True and I found a phrase and page number from the text
- ○ False and I can provide a counterexample
- ○ other

Check

Question **5**

Not complete

Points out of 7.00

Verify that det *AB* = det*A* det *B*, where $A = \begin{bmatrix} 3 & 0 \\ 6 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}$

What is *AB?*

What is determinant of *AB?*

What is the determinant of *A*?

What is the determinant of B?

Verify that det *AB* = det*A* det *B*

Check

Question **6**

Not complete

Points out of 5.00

Find the area of the parallelogram whose vertices are listed: (0,0), (5,2), (6,4), (11,6).

Specify the 2 vectors v_1, v_2 that emanate from the origin to make up the bases of the parallelogram.

Let v_1 be the vector with the smallest *x* value. What is v_1?

What is v_2?

What is the area of the parallelogram?

Check

Question **7**

Not complete

Points out of 1.00

Open
https://www.geogebra.org/m/syjzq7nt
Tibor Marcinek of Central Michigan University has created I, J, K, vertices of a parallelepiped and the vectors whose unit span make up the figure. The determinant of the corresponding matrix is also shown.

Leave I and K alone but drag the green J, on the right of the graph, around 3-space to change the figure, revealing the change in the determinant and volume of the parallelepiped. Solidify the visualization.

Next, drag J to the 2 on the red axis, on the bottom left side, but leave I and K as they originally were? Turn the figure to reveal the visualization better.

- the volume of the parallelepiped is 2
- the volume of the parallelepiped is 0
- other

Check

Question **8**

Not complete

Points out of 1.00

Find the volume of the parallelepiped with vectors $\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix}$

What is the volume of the parallelopiped?

[ ]

Check

Question **9**

Not complete

Points out of 1.00

Use the concept of volume to explain why the determinant of a 3x3 matrix A is zero if and only if A is not invertible.

Which of the following will help us here?

- ○ columns of A must be multiples
- ○ columns of A do not span $\mathbb{R}^3$
- ○ columns of A span $\mathbb{R}^3$
- ○ more than one of the above

Think about how to use the concept of volume to explain why the determinant of a 3x3 matrix A is zero if and only if A is not invertible. Can you write down a general argument?

○ yes ○ not yet

Check

Question **10**

Not complete

Points out of 1.00

This is part 2 of:

Use the concept of volume to explain why the determinant of a 3x3 matrix *A* is zero if and only if *A* is not invertible.

Examine the following argument:

First assume *A* is not invertible. The 3 column vectors don't span $\mathbb{R}^3$ via the negation of the statements of what makes a matrix invertible, so they are all either the origin, on a line through the origin, or in a plane through the origin. Regardless, they form a degenerate (smushed) parallelepiped, which has 0 volume, so 0 determinant.

Conversely assume a 3x3 matrix *A* has 0 determinant. The columns must form a 0 volume figure to create the 0 determinant, so the columns can't span all of $\mathbb{R}^3$. By the negation of the statements of what makes a matrix invertible, *A* is not invertible.

Is it a valid argument?

○ yes

○ no

Check

Question **11**

Not complete

Points out of 1.00

Determine if the set of vectors is a basis for $\mathbb{R}^3$: $\begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$

Examine the following arguments for their validity:

Argument 1: Compute the determinant of $A = \begin{bmatrix} 0 & 5 & 6 \\ 0 & 0 & 3 \\ -2 & 4 & 2 \end{bmatrix}$, which is the square matrix with the vectors as columns. The determinant is 0, so by the negation of what makes a matrix invertible theorem, the columns of A do not span $\mathbb{R}^3$ and are not linearly independent. So the vectors do not form a basis.

Argument 2: Compute the determinant of $A = \begin{bmatrix} 0 & 5 & 6 \\ 0 & 0 & 3 \\ -2 & 4 & 2 \end{bmatrix}$, which is the square matrix with the vectors as columns. The determinant is -30, which is nonzero, so by the what makes a matrix invertible theorem, the columns of A span $\mathbb{R}^3$ and are linearly independent. So the vectors form a basis.

Argument 3: Look at $A = \begin{bmatrix} 0 & 5 & 6 \\ 0 & 0 & 3 \\ -2 & 4 & 2 \end{bmatrix}$, which is the square matrix with the vectors as columns. Swap the rows of *A* (row 1 and 3, then row 2 with new row 3) so that *A* is row equivalent to a matrix in Gaussian with full row pivots (never inconsistent for $A\vec{x} = \vec{b}$) and full column pivots ($A\vec{x} = \vec{0}$ has only the trivial solution). So the vectors satisfy the definitions of span the entire space and linearly independent, and they form a basis

Argument 4: The matrix $A = \begin{bmatrix} 0 & 5 & 6 \\ 0 & 0 & 3 \\ -2 & 4 & 2 \end{bmatrix}$ is not invertible so by the negation of what makes a matrix invertible theorem, the columns of A do not span $\mathbb{R}^3$ and are not linearly independent. So the vectors do not form a basis.

Which, if any, arguments are valid here?

- ○ argument 1
- ○ argument 2
- ○ argument 3
- ○ argument 4
- ○ argument 1 and 4
- ○ argument 2 and 3
- ○ none of the above

Check

Question **12**

Not complete

Points out of 1.00

What is the significance of having a number associated to a matrix in the form of the determinant?

- ◯ It provides us a way to tell if a matrix is invertible (determinant nonzero) or not (determinant 0).
- ◯ Its absolute value has real-life meaning in area or volume which lead to a variety of applications!
- ◯ both
- ◯ neither

Check

Question **13**

Not complete

Points out of 3.00

Open
https://www.geogebra.org/m/uu9bw5dj
from Juan Carlos Ponce Campuzano, University of Queensland, Australia, which we saw previously. The vectors shown in the graph are the column vectors of the matrix, i.e. the image of the unit axes under the linear transformation.

New this time is that approximations of the determinant are shown below the matrix. Go through all the examples again to see how the determinant impacts the figure.

Example 1 is a dilation linear transformation. What does it do to the cube?

- ◯ shrinks each side by .5, making the cube 1/8 as big
- ◯ expands each side by 2, making the cube 4 times as big
- ◯ expands each side by 2, making the cube 8 times as big

In Example 2, what is the exact determinant

- ◯ .13
- ◯ .125
- ◯ other

In Example 5, turn the graph to see that the transformed cube has been squished into a smaller space, a plane in 3-space instead of retaining length, width, and height. Line up the transformed cube with the axes to see the squishing! What determinant is responsible for this squishing?

- ◯ negative determinant
- ◯ 0 determinant
- ◯ positive determinant

Check

Question **14**

Not complete

Points out of 1.00

To solidify and prepare for upcoming work, review and contemplate your knowledge and any questions that remain as related to definitions, concepts, computations, and examples from 3.1, 3.2, 3.3, including

- determinant computations: diagonal method or cofactor expansion method (Laplace expansion)
- determinant of a triangular matrix or a transpose of a matrix
- impact of row operations on determinants
- connection of determinants to invertibility and what makes a matrix invertible
- determinants as area of parallelogram or volume of parallelepiped and the impact of row operations

Since the material builds on itself, consider whether there is any material from before this module that you want to brush up on:

1.1

- algebra of linear equations: coefficients and variables
- geometry of linear equations in 2D and 3D: lines and planes
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions
- matrix of a linear system: coefficient matrix, augmented matrix, triangular form
- row equivalent systems
- algorithm for solving a linear system using elementary row operations of replacement, interchange, and scaling

1.2

- matrix of a linear system: row echelon form (Gaussian), reduced row echelon form (Gauss-Jordan)
- pivots: pivot position of a matrix, pivot column of a matrix
- row reduction algorithm we will most commonly use: elimination by forward phase and back substitution to row echelon form
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions with free variables and parametric solutions

1.3

- algebra of vectors: coordinates, addition of vectors, scalar multiplication of vectors, properties like associativity under addition (property ii on p. 29), a linear combination with weights, zero vector, span of a set of vectors=all the linear combinations, is a vector in the span?, vector equation $\longrightarrow$ augmented matrix
- geometry of vectors in 2D and 3D: directed segment, parallelogram for addition, on same line for scalar multiplication of vectors, origin=zero vector, a linear combination geometrically in the plane or 3D, span=all the linear combinations geometrically in the plane or 3D, spaces of subsets of R^n spanned by vectors

1.4

- algebra of matrix vector equation $A\vec{x} = \vec{b}$:
  - multiply a matrix and a column vector by linear combinations of the columns of *A* using weights from $\vec{x}$
  - span of the columns of *A* = set of all linear combinations of the columns of *A*
  - matrix vector equation $\longrightarrow$ vector equation $\longrightarrow$ augmented matrix
  - equations generic vectors $\vec{b}$ must satisfy to be in the span (Example 3)
  - dot products of rows of *A* with $\vec{x}$,
- geometry of solutions of matrix vector equation $A\vec{x} = \vec{b}$: spaces of subsets of R^3 spanned by the column vectors of A, geometry of such spaces (Figure 1)
- Theorem 4: relationship of consistency of $A\vec{x} = \vec{b}$ to always being a linear combination to spanning the entire R^m, where *m* is the number of rows, to having a pivot position in every row of *A*.
- identity matrix *I*

1.5

- algebra of homogeneous systems: $A\vec{x} = \vec{0}$
- algebraic solutions of homogenous systems always include the trivial solution=$\vec{0}$. nontrivial solutions, if any exist, are parameterized in parametric vector form using free variables to express those as well as the variables with pivots and then decomposed algebraically to showcase the algebra and geometry giving $t\vec{v}$ or $s\vec{u} + t\vec{v}$ or similar, where each free variable is attached to a vector.
- geometry of solutions of homogeneous systems are geometric spaces through the origin like lines, planes, or hyperplanes
- algebra of nonhomogeneous systens: $A\vec{x} = \vec{b}$

- solutions of nonhomogeneous systems in parametric vector form can be decomposed algebraically to showcase the algebra and geometry like $\vec{p} + t\vec{v}$, vectors ending on the line parallel to $\vec{v}$ or $\vec{p} + s\vec{u} + t\vec{v}$, vectors ending on the plane parallel to the one spanned by $\vec{u}, \vec{v}$...
- geometry of solutions of nonhomogeneous systems are geometric spaces translated away from the origin via adding $\vec{p}$

1.7

- linearly independent set of vectors and connection to a homogeneous equation having only the trivial solution
- linearly dependent set of vectors and connection to nontrivial solutions existing and providing a dependance relation
- geometry of linearly independent set of 2 vectors: independent directions in space versus along the same line (Figure 1)
- geometry of linearly independent set of 3 or more vectors: no one vector is in the span of the rest, i.e. they are all needed to span the space versus redundancy in the geometric space they span in the sense that they aren't all needed to generate the same space under linear combinations (Figure 2)
- linearly independent columns of a matrix
- redundancy of $\vec{0}$ in a set of vectors $\{\vec{v}_1 = \vec{0}, \vec{v}_2, \ldots \vec{v}_n\}$ (Theorem 9)

1.8 and 1.9

- linear transformation: addition and scalar multiplication
- left multiplication matrix representations
- dilation, projection, reflection, rotation, shear (see Examples 2-5 in 1.8, Examples 2-3 in 1.9, and tables 1-4 in 2.8)
- the algebraic image of the unit axes as a way to find the matrix of the transformation
- range of a linear transformation: the algebraic or geometric images or outputs, e.g. of the unit square as a way to visualize the transformation and understand its effects
- the range, image or output of a sheared sheep

2.1

- matrices: diagonal matrix [and main diagonal], zero matrix
- matrix operations: matrix addition, scalar multiplication of a matrix, matrix multiplication, powers of a matrix, left (or right) multiplication, transpose of a matrix
- matrix multiplication by linear combinations of the columns of A using weights from the corresponding column of B or by the dot products of a row of A with the corresponding column of B.
- algebraic properties that do hold for matrix multiplication: associativity and one-sided distributivity
- algebraic properties that don't hold for matrix multiplication: commutativity

2.2

- matrices: invertible (nonsingular) matrix, noninvertible (singular) matrix, elementary matrix
- determinant and inverse of a 2x2 matrix
- connection between invertibility and unique solutions
- inverse of a product of matrices and inverse of a transpose

2.3

- what makes a matrix invertible for a square matrix (Theorem 8 statements aside from f. and i., which we haven't covered)
- condition number (numerical note on p. 123)

2.7

- 2D and 3D computer graphics as columns of a matrix --- connect the dots!
- effects of 2D and 3D linear transformations from 1.8 and 1.9 and 3D transformations on figures--- algebra and matrix representation and geometry and visualization
- homogeneous coordinates
- composite transformations ABC is read right to left like functions, where C is the first action
- rotate about a point other than the origin (Figure 7)

2.8

- subspace properties: closed under addition and scalar multiplication
- spaces associated to a matrix: column space and null space
- basis: linearly independent spanning set
- basis for column space as the pivot columns
- basis for null space as the vectors attached to free variables in parametric solutions of the homogeneous system $A\vec{x} = \vec{0}$

2.9

- dimension of a space
- rank of a matrix (dimension of column space)
- nullity of a matrix (dimension of null space)
- rank nullity theorem (Theorem 14)
- what makes a matrix invertible continued: adding rank and nullity to Theorem 8 when the matrix is square

6.1

- inner product of $\vec{u}$ and $\vec{v}$ and connection to the dot product
- length or norm of a vector
- orthogonal vectors

When you have finished reviewing and reflecting, select one of the following (both receive full credit)

○ I currently have no questions

○ I will continue solidifying and understand that help is available in Dr. Sarah's more extensive feedback that follows below each question after I finish and open back up an entire practice quiz (this is more extensive than the hints that I can access during the open quiz), in Dr. Sarah's glossary/Wiki which is embedded into ASULearn from the linked terms, in Dr. Sarah's office hours and forum, and in Math Lab and Tutoring

Check

◄ 3.1, 3.2, 3.3 interactive video

Jump to...

second chance 3.1, 3.2, 3.3 practice quiz ►