5.1 and 5.2 Handwrite

Welcoming Environment: Actively listen to others and encourage everyone to participate! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Discuss and keep track of any questions your group has. Ask me questions during group work time as well as when I bring us back together. Try to help each other solidify and review the language of linear algebra, algebra, visualizations and intuition from this section, including those related to:

- eigenvector, eigenvalue, and eigenspace representations algebraically and input-output diagram
- eigenvectors and difference equations on p. 279
- nonzero eigenvalues and what makes a matrix invertible
- characteristic equation
- similar matrices via a similarity transformation (compare with what we did in 2.7 and computer graphics to rotate about a point other than the origin)
- application to dynamical systems

Take out your notes from the activities due today as well as both fill-in guides. Use them and each other to respond to the following by handwriting in the language of our class. Use only what we have covered so far in our readings, videos and quizzes.

1. **Building Community**: What are the preferred first names of those sitting near you? If you weren't able to be there, give reference to anyone you had help from or write N/A otherwise.

2. a) By hand, find the eigenvalues of $\begin{bmatrix} -4 & -1 \\ 6 & 1 \end{bmatrix}$. Show by hand work for the characteristic equation $|A - \lambda I| = 0$.

   b) By hand, find the eigenspace of A corresponding to $\lambda = -2$ (i.e. nullspace of $A - \lambda I$). Use strict Gaussian and back substitution and keep any variables without pivots free, like we usually do. Show all work, including the reduction instruction like $r_2' = 52r_1 + r_2$.

c) What does this $\lambda = -2$ tell us algebraically and geometrically about vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ on the line $y = -2x$ corresponding to the eigenspace of $t \begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix}$? For the algebraic perspective, fill in the blank: $A\vec{x} =$ __________$\vec{x}$. For the geometry, interpret this—explain below.

d) Apply this. Sketch an input vector and output vector **on one input-output diagram** using standard mathematical axes for the input eigenvector on the line $y = -2x$ with $\lambda = -2$ that we solved for from keeping any variables without pivots free and identify which is the input and which is the output.

3. Recall from the interactive video that for reflection across the $y = x$ line, vectors on the $y = x$ line had $\lambda = 1$ and vectors on the $y = -x$ line had $\lambda = -1$. Here, to extend this, imagine a general reflection matrix that reflects over a generic line in $\mathbb{R}^2$ and reason geometrically:

   a) If we are on a generic line of reflection that is fixed because it is the mirror of reflection, what is the eigenvalue?

   b) Are vectors on the line that is perpendicular to the line of reflection an eigenvector (i.e. do the outputs realign on the same line as the inputs)? If so, what is the eigenvalue?

   c) Aside from vectors on the line of reflection or perpendicular to it, are there any other eigenvectors for a general reflection matrix?

Next, as time allows before I bring us back together, work on the additional activities including any pollev activities and respond in your notes rather than here.

Help each other and PDF responses to ASULearn: If you are finished with the handwrite and additional activities before I bring us back together, first ensure that your entire group is finished too, and if not, help each other. Then submit your handwrite, continue reviewing and solidifying or discuss upcoming class work.

Collate your handwritten responses, preferably on this handout, into one full size multipage PDF for submission in the ASULearn assignment. I recommend you turn it in sometime today, but you have until the morning before the next class.