You can preview this quiz, but if this were a real attempt, you would be blocked because:

This quiz is not currently available

Question **1**

Not complete

Points out of 4.00

Open
https://www.geogebra.org/m/thu5sxs3
which we explored back in 1.3. Move the a slider around to actively engage with the impact of a scalar multiple $a = \lambda$ varying over the reals from -5..5 times the vector $\vec{u} = \begin{bmatrix}1\\4\\7\end{bmatrix}$, shown in the graph as $\lambda\vec{u}$. You can also turn the graph in 3-space to see different viewpoints. Drag the slider to see the scalar multiples and solidify the visualization.

Next, set the slider at $a = -.9$. How does $-.9\vec{u}$ compare to $\vec{u}$? $-.9\vec{u}$ is...

- ○on the same line through the origin
- ○on a different line through the origin

- ○pointed in the same direction
- ○pointed in a different direction

- ○the same length
- ○shorter
- ○longer

When eigenvectors exist, we can turn matrix multiplication of a vector to scalar multiplication of that vector. Why is this advantageous?

- ○scalar multiplication, where we scale a vector by λ, is faster than matrix multiplication and this can be important in computer applications
- ○scalar multiplication keeps vectors on the same line through the origin and this can be important in real-life applications and visualizations
- ○both of the above
- ○none of the above

Check

Question **2**

Not complete

Points out of 1.00

Is $\vec{x} = \begin{bmatrix} 11 \\ -3 \end{bmatrix}$ an eigenvector of $A = \begin{bmatrix} \frac{1}{3} & -2 \\ 1 & 1 \end{bmatrix}$? Multiply and see if it is a scaled version or not.

○ yes

○ no

If we look at the linear transformation corresponding to $\begin{bmatrix} \frac{1}{3} & -2 \\ 1 & 1 \end{bmatrix}$, does it keep $\begin{bmatrix} 11 \\ -3 \end{bmatrix}$ on the same line, the $y = \frac{-3}{11}x$ line?

○ yes

○ no

Check

Question **3**

Not complete

Points out of 7.00

For the geometry of eigenvalues and eigenvectors we look to see when the input vector is on the same line as the output vector, either pointing in the same direction (positive eigenvalue) or opposite direction (negative eigenvalue). The change in length of the vector is represented by the scalar eigenvalue, i.e. an eigenvalue of 4 means the length has stretched out to 4 times what it was while and eigenvalue of $\frac{1}{4}$ means the length has shrunk to 25% of what it was.

Open
https://www.geogebra.org/m/nfvyhewj

Example 1: Leave the sliders alone but drag the vector $\vec{u}$ via the endpoint shown there: drag it around the unit circle to look for eigenvalues and eigenvectors, where the output $\lambda\vec{u}$ is on the same line through the origin as is the input $\vec{u}$

What happens when $\vec{u}$ is on the y-axis?

- ○ it is an eigenvector since it lines up on the same line and the eigenvalue is 2 as the length of $\lambda\vec{u}$ is double the length of $\vec{u}$ and pointed in the same direction
- ○ it is an eigenvector since it lines up on the same line and the eigenvalue is -3 as the length of $\lambda\vec{u}$ is triple the length of $\vec{u}$ but pointed in the opposite direction
- ○ other

What happens when $\vec{u}$ is on the x-axis?

- ○ it is an eigenvector since it lines up on the same line and the eigenvalue is 2 as the length of $\lambda\vec{u}$ is double the length of $\vec{u}$ and pointed in the same direction
- ○ it is an eigenvector since it lines up on the same line and the eigenvalue is -3 as the length of $\lambda\vec{u}$ is triple the length of $\vec{u}$ but pointed in the opposite direction
- ○ other

Are there any other lines that are eigenvectors for this matrix?

- ○ yes there are other lines that realign
- ○ no, as all other lines, other than the x-axis and y-axis, has the input $\vec{u}$ making something other than a 0 or 180 degree angle with $\lambda\vec{u}$.

Example 2: Drag the sliders so that $a = 5, b = 5, c = 0, d = -5$. Then drag $\vec{u}$ around the circle. For this matrix, there is only one line that has eigenvectors.

Are vectors on the y-axis an eigenvector?

- ○ yes
- ○ no

Are vectors on the x-axis an eigenvector?

- ○ yes
- ○ no

Use the visualization to list what is an eigenvalue for this matrix?

[ ]

Example 3: Drag the sliders so that $a = 0, b = 1, c = 2, d = 3$. Then drag $\vec{u}$ around the circle to see when the vectors realign. Which is true?

- ○$\lambda = \pm 1$
- ○one line has λ negative but larger than -1 (i.e. flip while shrinking) and another line has λ larger than 1 (i.e. expanding in the same direction)
- ○two lines have λ larger than 1
- ○two lines have negative λ
- ○other

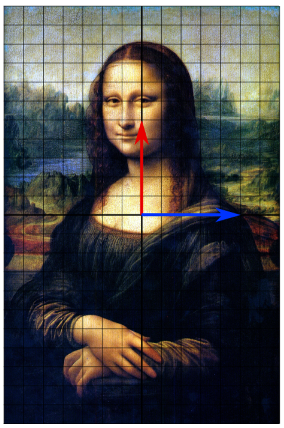
Check

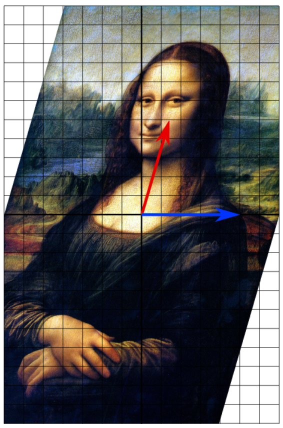
Question **4**

Not complete

Points out of 1.00

Consider this image of Mona Lisa horizontally sheared





What is an eigenspace of the sheared Mona Lisa?

- ○*x*-axis
- ○*y*-axis
- ○both
- ○neither

Check

Question **5**

Not complete

Points out of 1.00

For $A = \begin{bmatrix} -1 & 4 \\ 6 & 9 \end{bmatrix}$, is det $(A - (-3)I) = 0$? Compute to check.

○yes

○no

Is $\lambda = -3$ an eigenvalue of $A = \begin{bmatrix} -1 & 4 \\ 6 & 9 \end{bmatrix}$?

○yes

○no

Next, open
https://www.geogebra.org/m/ttvvhqrc
Drag the point at (1,0) at the end of the vector $\vec{u}$ around the unit circle to see if $\lambda = -3$ happens. Recall that eigenvalues tell us the scaling in $A\vec{u} = \lambda\vec{u}$ so we are looking to see when $\lambda\vec{u}$ realigns with $\vec{u}$ on the same line, and how we are stretched, shrunk, and/or flipped on that line.

Where does $\lambda = -3$ happen, if anywhere?

○does not exist for this matrix

○one line going through quadrants 1 and 3

○one line going through quadrants 2 and 4

○both of the above

○other

Check

Question **6**

Not complete

Points out of 1.00

Matrix algebra is useful in solving for eigenvalues and eigenvectors.

In order to find the eigenvalues of the square matrix *A* we want to find all scalars λ such that $A\vec{x} = \lambda\vec{x}$ for some non-zero vector $\vec{x}$. This is the same as finding all scalars λ such that the matrix equation $(A - \lambda I)\vec{x} = \vec{0}$ has nontrivial solutions.

By the theorem of what makes a matrix invertible in section 2.3, if we have a nontrivial solution of $(A - \lambda I)\vec{x} = \vec{0}$ then what does this say about invertibility of $A - \lambda I$?

○ $A - \lambda I$ is invertible

○ $A - \lambda I$ is singular or not invertible

What does this tell us about the determinant of $A - \lambda I$?

○ $|A - \lambda I| = 0$

○ $|A - \lambda I| \neq 0$

This last equation is called the characteristic equation and the solutions of λ to this equation are the eigenvalues of A. The characteristic equation is a polynomial of degree n, from the size of the $n \times n$ matrix and so there are at most n roots. For a 3×3 matrix, what is the largest number of eigenvalues we can have?

[ ]

Given λ: $A\vec{x} = 7\vec{x}$.
Subtract $7\vec{x}$ from both sides of the equation and apply the additive inverse to get $A\vec{x} - 7\vec{x} = \vec{0}$.
Bring in the identity matrix $7\vec{x} = 7I\vec{x}$ to substute in: $A\vec{x} - 7I\vec{x} = \vec{0}$ so that we'll be able to subtract matrices.
Then re-write this using distributivity properties as $(A\vec{x} - 7I)\,\vec{x} = \vec{0}$.
The non-zero solutions to $(A\vec{x} - 7I)\,\vec{x} = \vec{0}$ are the eigenvectors with eigenvalue 7.

Is this the null space or column space of $A - 7I$?

○ null space

○ column space

Check

Question **7**

Not complete

Points out of 4.00

Find the characteristic polynomial and the eigenvalues of the matrix $\begin{bmatrix} 9 & 3 \\ 3 & 9 \end{bmatrix}$.

The characteristic polynomial is (simplify)

λ^2 +

[ ]

λ +

[ ]

The smallest eigenvalue is:

[ ]

The largest eigenvalue is:

[ ]

Check

Question **8**

Not complete

Points out of 1.00

Which 3x3 matrix is singular, i.e. not invertible?

- ○ The one that has eigenvalues 1 and 7
- ○ The one that has eigenvalues -1, 0, 3
- ○ both
- ○ neither as they are both invertible

Check

Question **9**

Not complete

Points out of 1.00

Is the statement "A number *c* is an eigenvalue of *A* if and only if the equation $(A - cI)\vec{x} = \vec{0}$ has a nontrivial solution." true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

- ○ True and I found a phrase and page number from the text
- ○ False and I can provide a counterexample
- ○ other

Check

Question **10**

Not complete

Points out of 3.00

To find an eigenvector for the matrix $A = \begin{bmatrix} -1 & -6 \\ -4 & 4 \end{bmatrix}$ corresponding to $\lambda = 7$, if we solve for the nullspace of $(A - 7I)$, which variable is free if we choose the one corresponding to the missing pivot?

○first coordinate

○second coordinate

○both

Set the free variable as t. Then solve for the nullspace of $(A - 7I)$ corresponding to $\lambda = 7$ (simplify the coordinate that is not free):

[ ]
t
[ ]

How many vectors does a basis for the eigenspace corresponding to $\lambda = 7$ have?

[ ]

What is the geometry of the eigenspace corresponding to $\lambda = 7$?

○$\vec{0}$

○the line $y = -\frac{3}{4}x$

○the line $y = -\frac{4}{3}x$

○another line

○a plane

Next, open
https://www.geogebra.org/m/fftnrmg4
Drag the point at (1,0) at the end of the vector $\vec{u}$ around the unit circle to see if $\lambda = 7$ happens. Recall that eigenvalues tell us the scaling in $A\vec{u} = \lambda\vec{u}$ so we are looking to see when $\lambda\vec{u}$ realigns with $\vec{u}$ on the same line, and how we are stretched, shrunk, and/or flipped on that line.

Where does $\lambda = 7$ happen, if anywhere?

○does not exist for this matrix

○one line going through quadrants 1 and 3

○one line going through quadrants 2 and 4

○both of the above

○other

Check

Question **11**

Not complete

Points out of 13.00

If the augmented matrix for $A - 3I\vec{x} = \vec{0}$ reduces to $\begin{bmatrix} 1 & -1 & 6 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$,

which variables are free?

- ○ only x_1
- ○ only x_2
- ○ only x_3
- ○ x_1 and x_2
- ○ x_1 and x_3
- ○ x_2 and x_3

Solve for any variables with pivots that are not free in terms of the parameters. What is x_1?

- ○ *t*, or equivalent, as it is free
- ○ x_2
- ○ $-6x_3$
- ○ $-x_2 + 6x_3$
- ○ $x_2 - 6x_3$
- ○ other

Write the solutions in vector parameterized form and then factor out any free variables to write the eigenspace as a linear combination with the free variables as the weights.

If x_1 is factored out in the parameterization, what vector pairs with it?
Write n/a in all 3 spots if this variable has a pivot or write the vector otherwise.

[ ]

[ ]

[ ]

If x_2 is factored out in the parameterization, what vector pairs with it?
Write n/a in all 3 spots if this variable has a pivot or write the vector otherwise.

[ ]

[ ]

[ ]

If x_3 is factored out in the parameterization, what vector pairs with it?
Write n/a in all 3 spots if this variable has a pivot or write the vector otherwise.

Then solve for a basis for the eigenspace by taking the vectors from the linear combination. How many vectors does a basis for the eigenspace corresponding to $\lambda = 3$ have?

What is the geometry of the eigenspace corresponding to $\lambda = 3$?

- $\vec{0}$
- the line $y = -\frac{1}{6}x$
- another line
- a plane where this linear transformation acts as a dilation by 3

Check

Question **12**

Not complete

Points out of 1.00

Is the statement "Finding an eigenvector of *A* may be difficult, but checking whether a given vector is in fact an eigenvector is easy." true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

- True and I found a phrase and page number from the text
- False and I can provide a counterexample
- other

Check

Question **13**

Not complete

Points out of 1.00

Is the statement "To find the eigenvalues of A, reduce A to echelon form." true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

- ○ True and I found a phrase and page number from the text
- ○ False and I can provide a counterexample
- ○ other

Check

Question **14**

Not complete

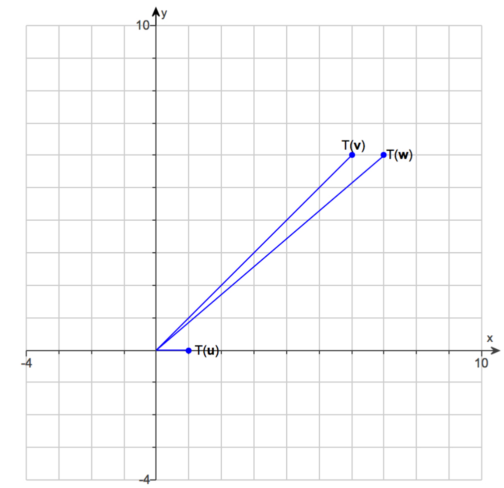
Points out of 1.00

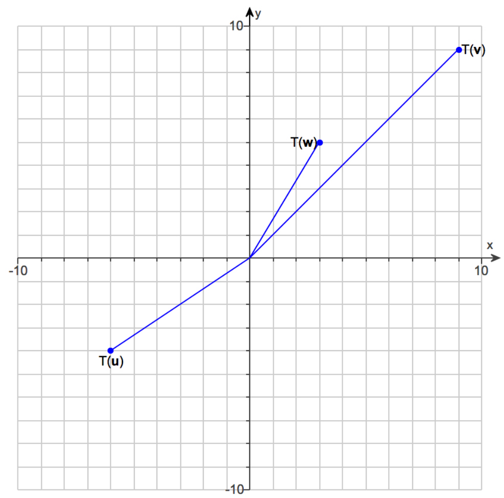
Assume A has two eigenvalues, $\lambda_u = -2$ corresponding to $\vec{u} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\lambda_v = 3$ corresponding to $\vec{v} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

Let T be the linear transformation given by $T(\vec{x}) = A\vec{x}$ for each $\vec{x}$.

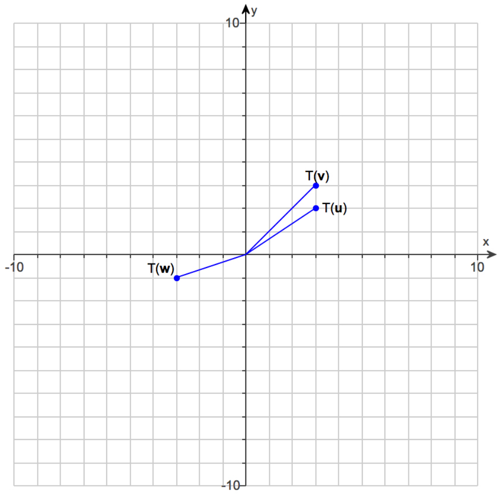
Let $\vec{w} = \vec{u} + \vec{v}$. Use the definition of eigenvalue as well as the linearity of the transformation to reason which graph represents $T(\vec{u}), T(\vec{v}), T(\vec{w})$?

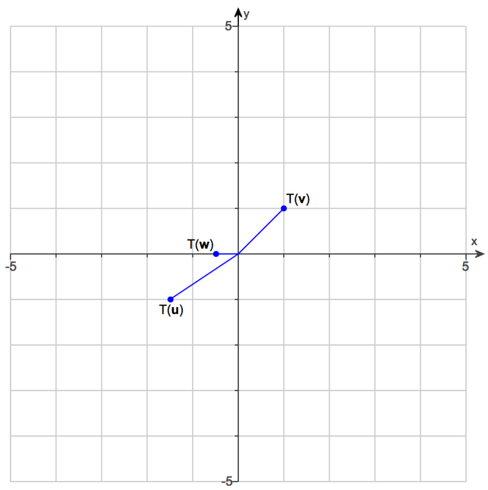
Select one:

○ 

○ 

○



○ 

Check

Question **15**

Not complete

Points out of 9.00

What are the eigenvalues of a triangular matrix?

○0

○the diagonal entries

○the entries off the diagonal

○other

What are the eigenvalues of $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 5 \\ 0 & 0 & 7 \end{bmatrix}$

List the eigenvalues **from smallest to largest**:

[ ]

,

[ ]

,

[ ]

.

For the geometry of the eigenvalues and eigenvectors of the vertical shear below, try to visualize in your head, but if you are stuck, you can open

https://www.geogebra.org/m/nfvyhewj

enter in an example of the transformation by changing the sliders, and then drag the vector $\vec{u}$ around the unit circle to see what stays on the same line through the origin and how it is scaled if it does.

For a vertical shear matrix, what stays on the same line through the origin under the linear transformation?

○only $\vec{0}$

○only the x-axis

○only the y-axis

○both the x-axis and the y-axis but no other line

○all of $\mathbb{R}^2$

What are the eigenvectors of a vertical shear matrix?

○only $\vec{0}$

○only the x-axis

○only the y-axis

○both the x-axis and the y-axis but no other line

○all of $\mathbb{R}^2$

How many vectors are in a basis for the eigenspace of a vertical shear matrix?

○0

○1

○2

○∞

What are the eigenvalues of a vertical shear matrix?

○-1

○0

○1

○both -1 and 1

○other

For the geometry of the eigenvalues and eigenvectors of the rotation by 180 degrees below, try to visualize in your head, but if you are stuck, you can open
https://www.geogebra.org/m/nfvyhewj
enter in the 180 degree rotation by changing the sliders, and then drag the vector $\vec{u}$ around the unit circle to see what stays on the same line through the origin and how it is scaled if it does.

For a rotation by 180 degrees, what stays on the same line through the origin under the linear transformation?

○only $\vec{0}$

○only the x-axis

○only the y-axis

○both the x-axis and the y-axis but no other line

○all of $\mathbb{R}^2$

What are the eigenvectors of rotation by 180 degrees?

○only $\vec{0}$

○only the x-axis

○only the y-axis

○both the x-axis and the y-axis but no other line

○all of $\mathbb{R}^2$

How many vectors are in a basis for the eigenspace of rotation by 180 degrees?

○0

○1

○2

○∞

What are the eigenvalues of rotation by 180 degrees?

○-1

○0

○1

○both -1 and 1

○other

Check

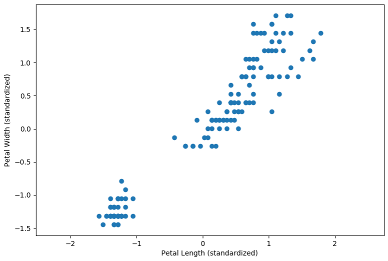
Question **16**

Not complete

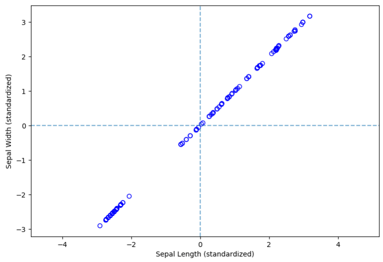
Points out of 1.00

In The why of principal component analysis – (ii) eigenvalues and eigenvectors, GodzillaButNicer

begins with this plot



and shows how the covariance matrix transforms the data



What are the eigenvectors, assuming that the points are now on a line through the origin?

- vectors on the line the data gets projected to as vectors that started there stay there
- vectors orthogonal to the line the data gets projected to as vectors that started there look like they get smushed to the origin
- both
- neither

In fact, the covariance matrix has pushed the points so that they lie *almost* on a line, the first principal component, which you can research further if you are interested in picking this up as a final project.

Check

Question **17**

Not complete

Points out of 1.00

To solidify and prepare for upcoming work, review and contemplate your knowledge and any questions that remain as related to definitions, concepts, computations, and examples from 5.1 and 5.2, including

- eigenvector, eigenvalue, and eigenspace
- eigenvectors and difference equations on p. 279
- nonzero eigenvalues and what makes a matrix invertible
- characteristic equation
- similar matrices via a similarity transformation (compare with what we did in 2.7 and computer graphics to rotate about a point other than the origin)
- application to dynamical systems

and consider 3.1, 3.2, 3.3, including

- determinant computations: diagonal method or cofactor expansion method (Laplace expansion)
- determinant of a triangular matrix or a transpose of a matrix
- impact of row operations on determinants
- connection of determinants to invertibility and what makes a matrix invertible
- determinants as area of parallelogram or volume of parallelepiped and the impact of row operations

Since the material builds on itself, consider whether there is any material from before this module that you want to brush up on:

1.1

- algebra of linear equations: coefficients and variables
- geometry of linear equations in 2D and 3D: lines and planes
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions
- matrix of a linear system: coefficient matrix, augmented matrix, triangular form
- row equivalent systems
- algorithm for solving a linear system using elementary row operations of replacement, interchange, and scaling

1.2

- matrix of a linear system: row echelon form (Gaussian), reduced row echelon form (Gauss-Jordan)
- pivots: pivot position of a matrix, pivot column of a matrix
- row reduction algorithm we will most commonly use: elimination by forward phase and back substitution to row echelon form
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions with free variables and parametric solutions

1.3

- algebra of vectors: coordinates, addition of vectors, scalar multiplication of vectors, properties like associativity under addition (property ii on p. 29), a linear combination with weights, zero vector, span of a set of vectors=all the linear combinations, is a vector in the span?, vector equation $\longrightarrow$ augmented matrix
- geometry of vectors in 2D and 3D: directed segment, parallelogram for addition, on same line for scalar multiplication of vectors, origin=zero vector, a linear combination geometrically in the plane or 3D, span=all the linear combinations geometrically in the plane or 3D, spaces of subsets of R^n spanned by vectors

1.4

- algebra of matrix vector equation $A\vec{x}=\vec{b}$:
  - multiply a matrix and a column vector by linear combinations of the columns of *A* using weights from $\vec{x}$
  - span of the columns of *A* = set of all linear combinations of the columns of *A*
  - matrix vector equation $\longrightarrow$ vector equation $\longrightarrow$ augmented matrix
  - equations generic vectors $\vec{b}$ must satisfy to be in the span (Example 3)
  - dot products of rows of *A* with $\vec{x}$,
- geometry of solutions of matrix vector equation $A\vec{x}=\vec{b}$: spaces of subsets of R^3 spanned by the column vectors of A, geometry of such spaces (Figure 1)
- Theorem 4: relationship of consistency of $A\vec{x}=\vec{b}$ to always being a linear combination to spanning the entire R^m, where *m* is the number of rows, to having a pivot position in every row of *A*.
- identity matrix *I*

1.5

- algebra of homogeneous systems: $A\vec{x} = \vec{0}$
- algebraic solutions of homogenous systems always include the trivial solution=$\vec{0}$. nontrivial solutions, if any exist, are parameterized in parametric vector form using free variables to express those as well as the variables with pivots and then decomposed algebraically to showcase the algebra and geometry giving $t\vec{v}$ or $s\vec{u} + t\vec{v}$ or similar, where each free variable is attached to a vector.
- geometry of solutions of homogeneous systems are geometric spaces through the origin like lines, planes, or hyperplanes
- algebra of nonhomogeneous systens: $A\vec{x} = \vec{b}$
- solutions of nonhomogeneous systems in parametric vector form can be decomposed algebraically to showcase the algebra and geometry like $\vec{p} + t\vec{v}$, vectors ending on the line parallel to $\vec{v}$ or $\vec{p} + s\vec{u} + t\vec{v}$, vectors ending on the plane parallel to the one spanned by $\vec{u}, \vec{v}$...
- geometry of solutions of nonhomogeneous systems are geometric spaces translated away from the origin via adding $\vec{p}$

1.7

- linearly independent set of vectors and connection to a homogeneous equation having only the trivial solution
- linearly dependent set of vectors and connection to nontrivial solutions existing and providing a dependance relation
- geometry of linearly independent set of 2 vectors: independent directions in space versus along the same line (Figure 1)
- geometry of linearly independent set of 3 or more vectors: no one vector is in the span of the rest, i.e. they are all needed to span the space versus redundancy in the geometric space they span in the sense that they aren't all needed to generate the same space under linear combinations (Figure 2)
- linearly independent columns of a matrix
- redundancy of $\vec{0}$ in a set of vectors $\{\vec{v}_1 = \vec{0}, \vec{v}_2, \ldots \vec{v}_n\}$ (Theorem 9)

1.8 and 1.9

- linear transformation: addition and scalar multiplication
- left multiplication matrix representations
- dilation, projection, reflection, rotation, shear (see Examples 2-5 in 1.8, Examples 2-3 in 1.9, and tables 1-4 in 2.8)
- the algebraic image of the unit axes as a way to find the matrix of the transformation
- range of a linear transformation: the algebraic or geometric images or outputs, e.g. of the unit square as a way to visualize the transformation and understand its effects
- the range, image or output of a sheared sheep

2.1

- matrices: diagonal matrix [and main diagonal], zero matrix
- matrix operations: matrix addition, scalar multiplication of a matrix, matrix multiplication, powers of a matrix, left (or right) multiplication, transpose of a matrix
- matrix multiplication by linear combinations of the columns of A using weights from the corresponding column of B or by the dot products of a row of A with the corresponding column of B.
- algebraic properties that do hold for matrix multiplication: associativity and one-sided distributivity
- algebraic properties that don't hold for matrix multiplication: commutativity

2.2

- matrices: invertible (nonsingular) matrix, noninvertible (singular) matrix, elementary matrix
- determinant and inverse of a 2x2 matrix
- connection between invertibility and unique solutions
- inverse of a product of matrices and inverse of a transpose

2.3

- what makes a matrix invertible for a square matrix (Theorem 8 statements aside from f. and i., which we haven't covered)
- condition number (numerical note on p. 123)

2.7

- 2D and 3D computer graphics as columns of a matrix --- connect the dots!
- effects of 2D and 3D linear transformations from 1.8 and 1.9 and 3D transformations on figures--- algebra and matrix representation and geometry and visualization
- homogeneous coordinates
- composite transformations ABC is read right to left like functions, where C is the first action
- rotate about a point other than the origin (Figure 7)

2.8

- subspace properties: closed under addition and scalar multiplication
- spaces associated to a matrix: column space and null space
- basis: linearly independent spanning set
- basis for column space as the pivot columns
- basis for null space as the vectors attached to free variables in parametric solutions of the homogeneous system $A\vec{x} = \vec{0}$

2.9

- dimension of a space
- rank of a matrix (dimension of column space)
- nullity of a matrix (dimension of null space)
- rank nullity theorem (Theorem 14)
- what makes a matrix invertible continued: adding rank and nullity to Theorem 8 when the matrix is square

6.1

- inner product of $\vec{u}$ and $\vec{v}$ and connection to the dot product
- length or norm of a vector
- orthogonal vectors

When you have finished reviewing and reflecting, select one of the following (both receive full credit)

○ I currently have no questions

○ I will continue solidifying and understand that help is available in Dr. Sarah's more extensive feedback that follows below each question after I finish and open back up an entire practice quiz (this is more extensive than the hints that I can access during the open quiz), in Dr. Sarah's glossary/Wiki which is embedded into ASULearn from the linked terms, in Dr. Sarah's office hours and forum, and in Math Lab and Tutoring

Check

◄ 5.1 and 5.2 interactive video

Jump to...

second chance 5.1 and 5.2 practice quiz ►