

5.6 Handwrite Practice

Handwrite 1. and collate into a single multi-page PDF for submission in ASULearn.

1. Deep in the redwood forests of California, dusky-footed wood rats provide up to 80% of the diet for the spotted owl, the main predator of the wood rat. Admittedly, the model is unrealistic in several respects, but it can provide a starting point for the study of nonlinear models used by environmental scientists.
 - a) Suppose the predator-prey matrix for these two population is given as $\begin{bmatrix} .5 & .4 \\ -p & 1.1 \end{bmatrix}$ with x as owls and y as wood rats. We examine the predation parameter $p = .2$. In Maple, execute `with(LinearAlgebra): with(plots): A:=Matrix([[5/10, 4/10], [-2/10, 11/10]]): Eigenvectors(A);` What are the eigenvalues and eigenvectors?
 - b) Explain why the eigenvectors span all of \mathbb{R}^2 using either an argument using determinant and the what makes a matrix invertible theorem or the definition of span by augmenting with a generic vector and reducing. Show work and provide reasoning.
 - c) Write out the eigenvector decomposition for \vec{x}_k for this system.
 - d) Do the populations grow, die off, stabilize, or exhibit some other behavior in the long run for most starting positions? Annotate how you can tell from the eigenvector decomposition.
 - e) What is the eventual ratio of the populations the system tends to in the long run for most starting positions? Annotate how you can tell from the eigenvector decomposition.
 - f) For most starting positions, what is the yearly rate of change (growth rate of change, die off rate of change, or stability rate of change) in the long run? Annotate how you can tell from the eigenvector decomposition.
 - g) Sketch a by-hand plot of the two eigenvectors, select a starting position in the 1st quadrant that is not on either eigenvector and demonstrate the long-term behavior in a trajectory diagram.