5.6 Handwrite

Welcoming Environment: Actively listen to others and encourage everyone to participate! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Discuss and keep track of any questions your group has. Ask me questions during group work time as well as when I bring us back together. Try to help each other solidify and review the language of linear algebra, algebra, visualizations and intuition from this section, including those related to:

- eigenvector decomposition
- longterm behavior or steady state
- geometry of solutions and a trajectory of the dynamical system
- relationship of the magnitude (absolute value) of the eigenvalues and the longterm behavior, including which term is dominant in the longterm and what that means for trajectories

Take out your notes from the activities due today as well as both fill-in guides. Use them and each other to respond to the following by handwriting in the language of our class. Use only what we have covered so far in our readings, videos and quizzes.

1. **Building Community**: What are the preferred first names of those sitting near you? If you weren't able to be there, give reference to anyone you had help from or write N/A otherwise.

2. Deep in the redwood forests of California, dusky-footed wood rats provide up to 80% of the diet for the spotted owl, the main predator of the wood rat. Admittedly, the model is unrealistic in several respects, but it can provide a starting point for the study of nonlinear models used by environmental scientists.

   a) Suppose the predator-prey matrix for these two population is given as $\begin{bmatrix} .5 & .4 \\ -p & 1.1 \end{bmatrix}$ with x as owls and y as wood rats. We examine the predation parameter p =.2. In Maple, we can execute packages and with(LinearAlgebra): with(plots): A:=Matrix([[5/10, 4/10], [-2/10, 11/10]]): Eigenvectors(A); to obtain $\begin{bmatrix} \frac{9}{10} \\ \frac{7}{10} \end{bmatrix}$ $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$. For each eigenvalue, what is its corresponding Maple eigenvector and eigenspace?

   $\lambda = \frac{9}{10}$ eigenvector: eigenspace: $\lambda = \frac{7}{10}$ eigenvector: eigenspace:

   b) Sketch an input-output diagram on one graph using standard mathematical axes and the input as Maple's eigenvector corresponding to $\lambda = \frac{7}{10}$ and identify which is the input and which is the output.

   c) Explain why Maple's eigenvectors (when taken together) span all of $\mathbb{R}^2$ using either an argument using determinant and the what makes a matrix invertible theorem or the definition of span by augmenting with a generic vector and reducing. Show work and provide reasoning.

d) Write out our eigenvector decomposition for $\vec{x}_k = a_1\lambda_1^k\vec{v}_1 + a_2\lambda_2^k\vec{v}_2$ but filled in with the relevant numbers for the eigenvalues and eigenvectors for this system.

e) Do the populations grow, die off, stabilize, or exhibit some other behavior in the long run for most starting positions? Annotate how you can tell from the eigenvector decomposition.

f) What is the eventual ratio of the populations the system tends to in the long run for most starting positions? Annotate how you can tell from the eigenvector decomposition.

g) For most starting positions, what is the yearly rate of change (growth rate, die off rate, or stability rate) in the long run? Annotate how you can tell from the eigenvector decomposition.

h) Sketch a by-hand plot of Maple's eigenvectors, using our standard mathematical axes, select a starting position in the 1st quadrant that is not on either eigenvector and demonstrate the long-term behavior in a trajectory diagram.

Next, as time allows before I bring us back together, work on the additional activities including any pollev activities and respond in your notes rather than here.

Help each other and PDF responses to ASULearn: If you are finished with the handwrite and additional activities before I bring us back together, first ensure that your entire group is finished too, and if not, help each other. Then submit your handwrite, continue reviewing and solidifying or discuss upcoming class work.

Collate your handwritten responses, preferably on this handout, into one full size multipage PDF for submission in the ASULearn assignment. I recommend you turn it in sometime today, but you have until the morning before the next class.