

Question 1

Not complete

Points out of 1.00

In order to write the [eigenvector decomposition](#), we need the [eigenvectors](#) to [span](#) the space we are working in so that any initial condition can be expressed in terms of them. If the [Eigenvectors](#) command of a matrix outputs in Maple as $\begin{bmatrix} \frac{9}{10} & \\ \frac{7}{10} & \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$, then examine the following arguments for their validity:

Argument 1: The [determinant](#) of the matrix of [eigenvectors](#) is $ad - bc = 1 \cdot 1 - 2 \cdot 1 = -1$, which is not zero, so by the [what makes a matrix invertible](#) theorem the columns of the matrix [span](#) all of \mathbb{R}^2

Argument 2: The definition of [span](#) is the set of all [linear combinations](#) so we set up a [generic vector](#) $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. The corresponding [augmented matrix](#) for the system is $\begin{bmatrix} 1 & 2 & b_1 \\ 1 & 1 & b_2 \end{bmatrix}$. Reducing using [Gaussian](#), we see we have full row [pivots](#): $\xrightarrow{r'_2 = -r_1 + r_2}$

$$\begin{bmatrix} 1 & 2 & b_1 \\ 0 & -1 & -b_1 + b_2 \end{bmatrix}$$

Thus we are never inconsistent and so every $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ [vector](#) in \mathbb{R}^2 is in the [span](#) of the [eigenvectors](#).

Argument 3: The [span](#) is -1 which is not 0 so we [span](#) all of \mathbb{R}^2

Which is true:

- Only Argument 1 is valid
- Only Argument 2 is valid
- Only Argument 3 is valid
- Argument 1 and Argument 2 are valid
- Argument 1 and Argument 3 are valid
- Argument 2 and Argument 3 are valid
- All three arguments are valid

Is "The [vectors](#) are [linearly independent](#)" sufficient to explain why [eigenvector](#) representatives [span](#) an entire space?

- yes
- no

Check

Question 2

Not complete

Points out of 7.00

One reason Calculus II with analytic geometry is a prerequisite for linear algebra is that in that class we apply limits to diverse objects including improper integrals and partial sums of series. The key in this section is applying limits to numbers that are [coefficients](#) within a [linear combination](#) of [vectors](#). In linear algebra, our time steps are discrete rather than continuous (like once a day or year).

What is $\lim_{k \rightarrow \infty} \left(\frac{2}{3}\right)^k$

- 0
- $\frac{2}{3}$
- ∞
- DNE
- other

What happens in the [longterm](#) as k grows large to $a_1 \left(\frac{2}{3}\right)^k \begin{bmatrix} 1 \\ 5 \end{bmatrix}$? What does this tend to?

- 0
- $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- $\frac{2}{15}$
- $\begin{bmatrix} a_1 \frac{2}{3} \\ a_1 \frac{10}{3} \end{bmatrix}$
- ∞
- $\begin{bmatrix} \infty \\ \infty \end{bmatrix}$
- DNE
- other

What is $\lim_{k \rightarrow \infty} \left(\frac{3}{2}\right)^k$

- 0
- $\frac{3}{2}$
- ∞
- DNE
- other

What happens in the [longterm](#) as k grows large to $a_2 \left(\frac{3}{2}\right)^k \begin{bmatrix} 5 \\ 6 \end{bmatrix}$? What does this tend to?

- 0
- $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- $\frac{15}{12}$

$\begin{bmatrix} a_2 \frac{15}{2} \\ a_2 9 \end{bmatrix}$

∞

$\begin{bmatrix} \infty \\ \infty \end{bmatrix}$

DNE

other

What is $\lim_{k \rightarrow \infty} 1^k$

0

1

∞

DNE

other

What happens in the [longterm](#) as k grows large to $a_3 1^k \begin{bmatrix} 5 \\ 6 \end{bmatrix}$? What does this tend to?

0

$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\frac{5}{6}$

$\begin{bmatrix} 5a_3 \\ 6a_3 \end{bmatrix}$

∞

$\begin{bmatrix} \infty \\ \infty \end{bmatrix}$

DNE

other

In the cases of growth and die off, the [slope](#) of the [vector](#) tells us how we die off or grow and what [line](#) we are asymptotic to. What is the [line](#) we are asymptotic to in $a_2 \left(\frac{3}{2}\right)^k \begin{bmatrix} 5 \\ 6 \end{bmatrix}$?

$y = \frac{5}{6}x$

$y = \frac{6}{5}x$

In dynamical systems where we have sums like $a_1 \left(\frac{2}{3}\right)^k \begin{bmatrix} 1 \\ 5 \end{bmatrix} + a_2 \left(\frac{3}{2}\right)^k \begin{bmatrix} 5 \\ 6 \end{bmatrix}$, the dominant piece in the limit determines [longterm behavior](#), i.e. whichever power of k is largest magnitude (i.e. largest absolute value)!

Check

Question 3

Not complete

Points out of 6.00

Open

<https://www.geogebra.org/m/xtvtxz6u>

The system shown has [eigenvectors](#) on the x -axis as well as the y -axis and no other [eigenvectors](#).

Drag the a and b sliders, which are the coordinates of the initial condition in the first quadrant, in order to see how the [trajectory](#) changes as k gets large, up to $k = 50$ here.

What happens in the long run for most starting positions as you move the sliders?

- die off to the origin: the dominant [eigenvalue](#) has absolute value less than 1
- going to stability towards a fixed [vector](#), one that is away from the origin: the dominant [eigenvalue](#) is 1
- infinite growth: the dominant [eigenvalue](#) has absolute greater than 1

What axis does the [trajectory](#) go asymptotic to (for die off or growth) or go towards (for stability) in the long run for most starting positions as you move the sliders?

- x -axis
- y -axis

Next, open the second [trajectory](#)

<https://www.geogebra.org/m/ynx9eaw8>

The system shown also has [eigenvectors](#) on the x -axis as well as the y -axis and no other [eigenvectors](#).

Drag the a and b sliders. What happens in the long run in the second [trajectory](#) for most starting positions as you move the sliders?

- die off to the origin: the dominant [eigenvalue](#) has absolute value less than 1
- going to stability towards a fixed [vector](#), one that is away from the origin: the dominant [eigenvalue](#) is 1
- infinite growth: the dominant [eigenvalue](#) has absolute greater than 1

What axis does the [trajectory](#) go asymptotic to (for die off or growth) or go towards (for stability) in the long run for most starting positions in [trajectory](#) 2 as you move the sliders?

- x -axis
- y -axis

Finally, open

<https://www.geogebra.org/m/ayvfsgyx>

The system shown also has [eigenvectors](#) on the x -axis as well as the y -axis and no other [eigenvectors](#).

Drag the a and b sliders. What happens in the long run in the third [trajectory](#) for most starting positions as you move the sliders?

- die off to the origin: the dominant [eigenvalue](#) has absolute value less than 1
- going to stability towards a fixed [vector](#), one that is away from the origin: the dominant [eigenvalue](#) is 1
- infinite growth: the dominant [eigenvalue](#) has absolute greater than 1

What axis does the [trajectory](#) go asymptotic to (for die off or growth) or go towards (for stability) in the long run for most starting positions as you move the sliders?

- x -axis
- y -axis

Check

Question 4

Not complete

Points out of 2.00

Examine the following 3 [trajectory](#) diagrams, which show the animated [trajectory](#) of the path of an initial condition that begins in quadrant 1 but doesn't start on either [eigenvector](#). In your notes, roughly sketch these and label them from among growth, die off, stability (which is which). The animated red +'s show the [trajectory](#) progressing and in the long run.

Image 1:

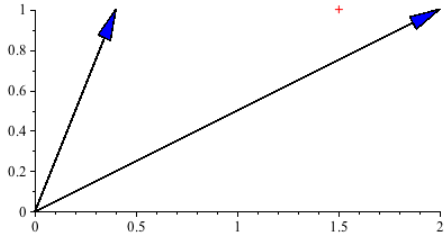


Image 2:

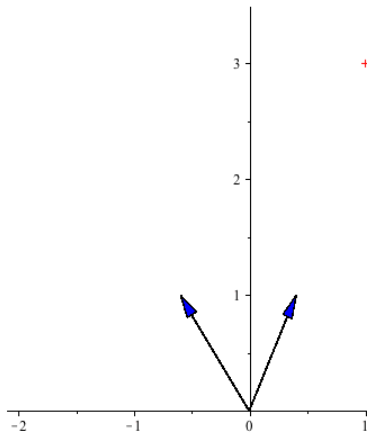
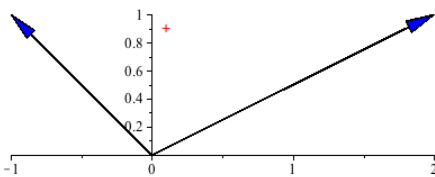


Image 3:



Which image corresponds to the situation where the dominant [eigenvalue](#) $|\lambda| < 1$?

Image 1

Image 2

- Image 3
- none of the above

Which image corresponds to the situation where the dominant [eigenvalue](#) $\lambda = 1$

- Image 1
- Image 2
- Image 3
- none of the above

Which image corresponds to the situation where the dominant [eigenvalue](#) $|\lambda| > 1$

- Image 1
- Image 2
- Image 3
- none of the above

For a linear system with real [eigenvalues](#), in the long run can one population grow along one [eigenvector](#) while the other dies off along another?

- yes, that is possible
- that's impossible in the long run as the dominant [eigenvalue](#) controls the behavior of the system in the long run and forces it to tend to the corresponding [eigenvector](#) in the long run, with the [eigenvalue](#) telling us whether the multiple populations die off, grow, or stabilize asymptotic to or towards that [eigenvector](#)

Check

Question 5

Not complete

Points out of 7.00

Let A be a 2x2 matrix with [eigenvalues](#)

$\lambda_1 = 3$ corresponding to the [eigenvector](#) $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and

$\lambda_2 = 1/3$ corresponding to the [eigenvector](#) $\vec{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

Let a_i be the [basis coefficient](#) of \vec{v}_i that represents the initial condition.

Write the [eigenvector decomposition](#) for the system \vec{x}_k :

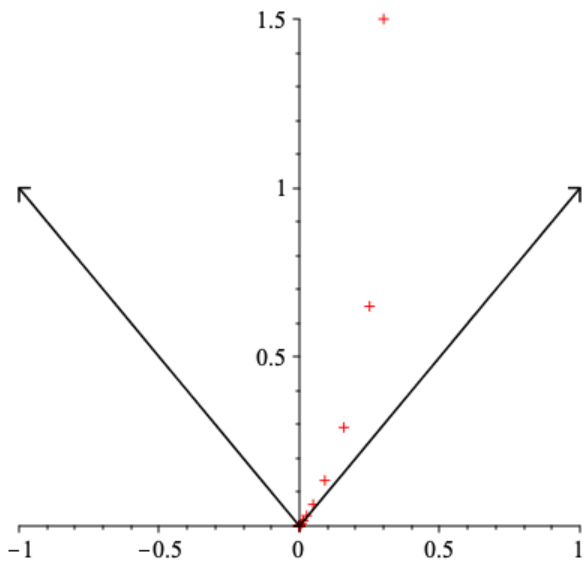
	k		k		
a_1	<input type="text"/>	<input type="text"/>	$+a_2$	<input type="text"/>	<input type="text"/>
	<input type="text"/>	<input type="text"/>		<input type="text"/>	<input type="text"/>

What is the mathematical [longterm behavior](#) for most initial conditions?

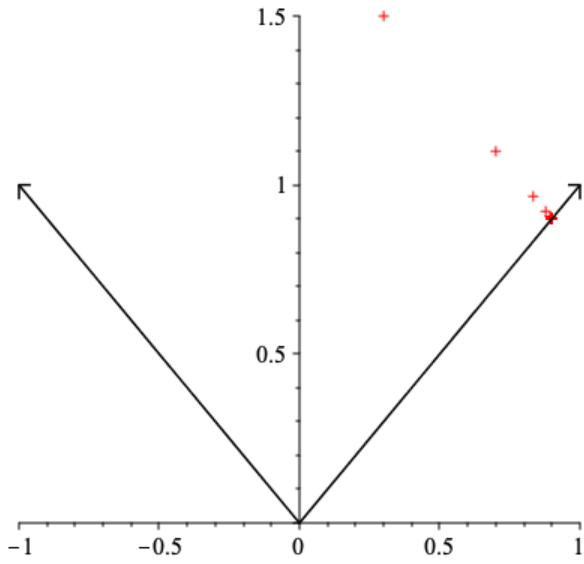
- dies out along $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
- stabilizes along $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
- grows along $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
- dies out along $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$
- stabilizes along $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$
- grows along $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$
- other

Which [trajectory](#) graph could represent this system based on the above decomposition?

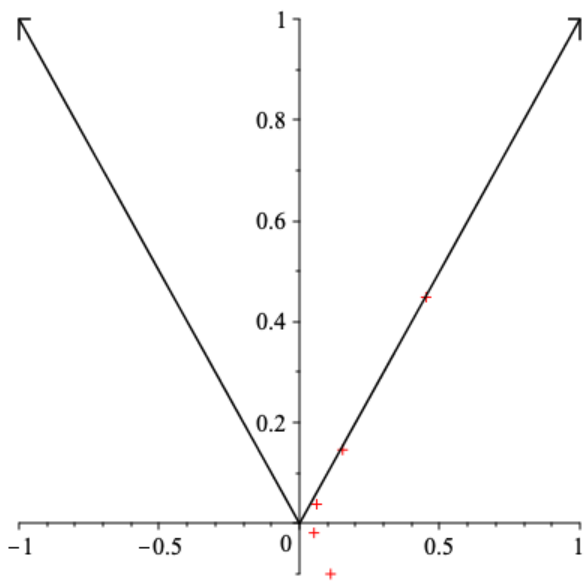
graph 1



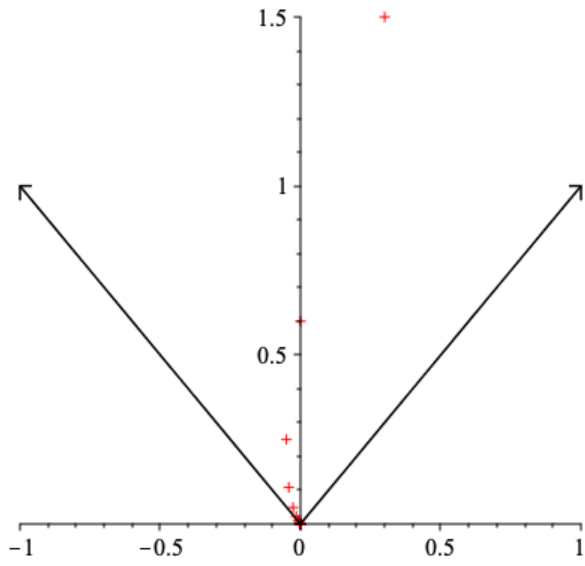
graph 2



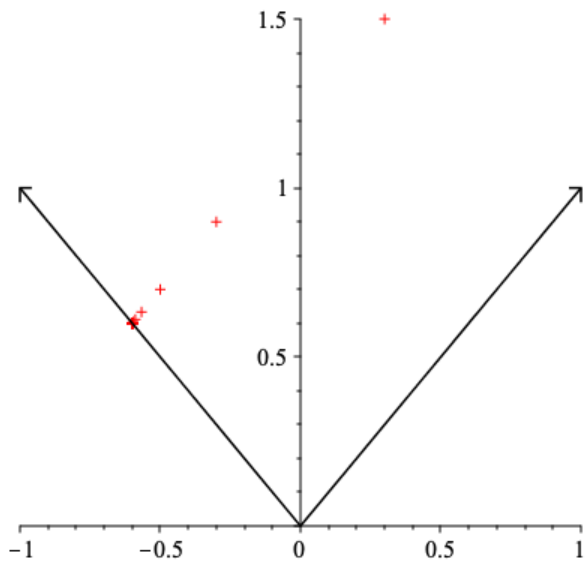
graph 3



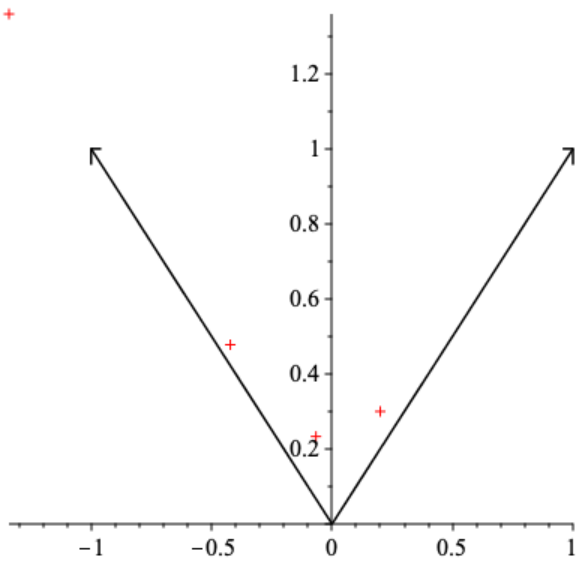
graph 4



graph 5



graph 6



- graph 1
- graph 2
- graph 3
- graph 4
- graph 5
- graph 6
- other

If we had an [eigenvalue](#) of 1, then we have a decomposition like

$$a_1 1^k \begin{bmatrix} a_2 \\ a_3 \end{bmatrix} + b_1 \lambda^k \begin{bmatrix} b_2 \\ b_3 \end{bmatrix} \text{ where } |\lambda| < 1, \text{ as it is the weaker } \text{eigenvalue}.$$

In this case, because $1^k = 1$, we can reduce and combine the constants into the [eigenvectors](#) so that it looks like

$$\begin{bmatrix} a_1 a_2 \\ a_1 a_3 \end{bmatrix} + \lambda^k \begin{bmatrix} b_1 b_2 \\ b_1 b_3 \end{bmatrix}$$

which is a constant [vector](#) + t another constant [vector](#).

This should look familiar from chapter 1 work we did on objects like $\vec{u} + t\vec{v}$.

Which graphs have an [eigenvalue](#) of 1?

- graphs 1 and 4
- graphs 2 and 5
- graphs 3 and 6

Check

Question 6

Not complete

Points out of 1.00

Let A be a 2x2 matrix with [eigenvalues](#)

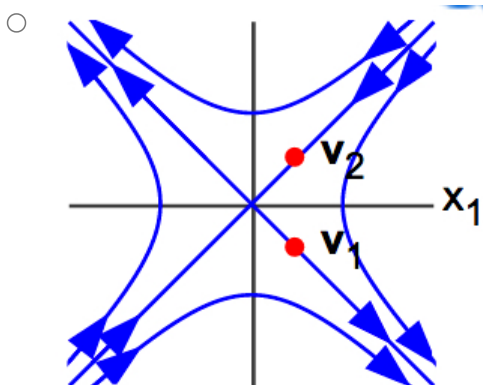
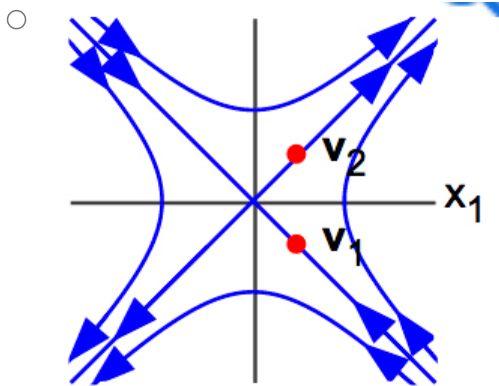
$\lambda_1 = 1/3$ corresponding to the [eigenvector](#) $\vec{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and

$\lambda_2 = 3$ corresponding to the [eigenvector](#) $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Sketch a [trajectory](#) diagram for the [longterm behavior](#) of the system, by graphing the two [eigenvectors](#) and picking a starting point in the first quadrant to sketch what happens over time.

While the [trajectory](#) diagram will look different than the visualizations below, since yours will just show 1 [trajectory](#) within the first quadrant, compare your [trajectory](#) diagram with the diagrams below. Below they show what happens if we start on either of the [lines](#) corresponding to the [eigenvectors](#), as well as what happens in each quadrant off the [eigenvector lines](#) over time. Which diagram represents the system's long-term behavior?

Select one:



other

Check

Question 7

Not complete

Points out of 13.00

Let A be a 3×3 matrix with eigenvalues $\lambda_1 = 4$ corresponding to the eigenvector $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$,

$\lambda_2 = 3/4$ corresponding to the eigenvector $\vec{v}_2 = \begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}$, and

$\lambda_3 = 1/4$ corresponding to the eigenvector $\vec{v}_3 = \begin{bmatrix} -2 \\ -4 \\ 8 \end{bmatrix}$

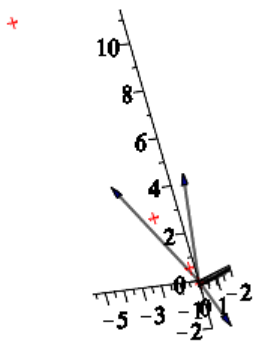
Let a_i be the basis coefficient of \vec{v}_i that represents the initial condition.

Write the eigenvector decomposition for the system \vec{x}_k using the ordering from above:

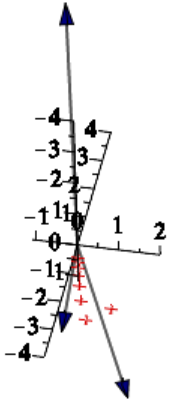
k		k		k	
a_1			$+a_2$		

Consider the following three graphs in 3-space

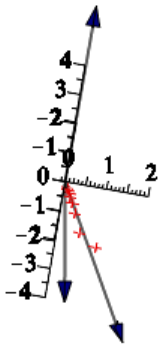
graph 1 shows growth in the longterm asymptotic to $\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$



graph 2 shows die off in the longterm asymptotic to $\begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}$



graph 3 shows die off in the [longterm](#) asymptotic to $\begin{bmatrix} -2 \\ -4 \\ 8 \end{bmatrix}$



What happens to the system as k gets large for most initial conditions?

- grows along $\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ like in graph 1
- dies off along $\begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}$ like in graph 2
- dies off along $\begin{bmatrix} -2 \\ -4 \\ 8 \end{bmatrix}$ like in graph 3
- other

Check

Question 8

Not complete

Points out of 5.00

Say we have a system of Foxes (x) and Rabbits (y), where the matrix A represents the change in populations from year to year, and the largest magnitude [eigenvalue](#) for A is 1.1 and the corresponding [eigenvector](#) is $\begin{bmatrix} 2 \\ 100 \end{bmatrix}$

Part a) The foxes grow in the long run by

% each year, for most starting positions. In other words, what is the rate of change, since that is the growth rate.

Part b) Do the rabbits and foxes change at the same rate (as each other) in the long run?

yes

no

Part c) What ratio do the populations tend to in the [longterm](#), for most starting positions? (Do NOT simplify).

foxes to

rabbits

Part d) What is the equation of the [line](#) the populations tend to in the [longterm](#), for most starting positions?

$y =$

x

Check

Question 9

Not complete

Points out of 5.00

Let A be a matrix with [eigenvector decomposition](#) $\vec{x}_k = a_1\lambda_1^k \begin{bmatrix} a \\ b \end{bmatrix} + a_2\lambda_2^k \begin{bmatrix} c \\ d \end{bmatrix}$.

Fill in the following to see the matrix algebra for \vec{x}_{k+1}

$\vec{x}_{k+1} =$

\vec{x}_k .

Substitute for \vec{x}_k and write out the steps to show that this equals $a_1\lambda_1^{k+1} \begin{bmatrix} a \\ b \end{bmatrix} + a_2\lambda_2^{k+1} \begin{bmatrix} c \\ d \end{bmatrix}$?

Which algebraic properties are needed?

Should we apply the [inverse](#)?

yes

no

Should we use left distributivity?

yes

no

Should we pull scalars through a matrix?

yes

no

Should we use the definition of [eigenvector](#)?

yes

no

Check

Question 10

Not complete

Points out of 2.00

A basketball team has a 60% probability of winning their next game if they have won their previous game but only a 30% probability of winning their next game if they have lost their previous game. Let x be % chance of winning game k and y be the % chance of losing that game.

What is the probability of winning the next game?

- $.6x + .3y$
- $.4x + .7y$

If the [eigenvector decomposition](#) for the system is

$$\left(\frac{1}{7}\right) (1^k) \begin{bmatrix} 3 \\ 4 \end{bmatrix} + \left(\frac{-4}{7}\right) (0.3^k) \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

then because $1^k = 1$, we can reduce as follows:

$$= \left(\frac{1}{7}\right) \begin{bmatrix} 3 \\ 4 \end{bmatrix} + \left(\frac{-4}{7}\right) (0.3^k) \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

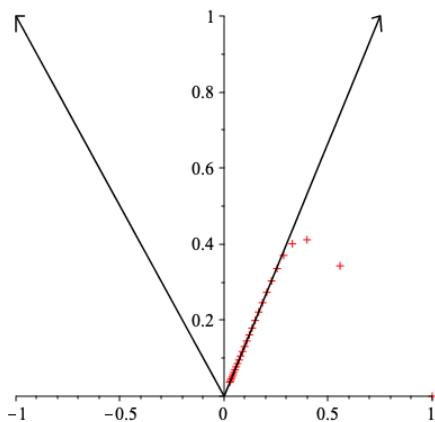
$$= \begin{bmatrix} \frac{3}{7} \\ \frac{4}{7} \end{bmatrix} + \left(\frac{-4}{7}\right) (0.3^k) \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Notice that the [basis coefficients](#) are specific numbers here, for a specific initial condition. What happens to the system in the long run?

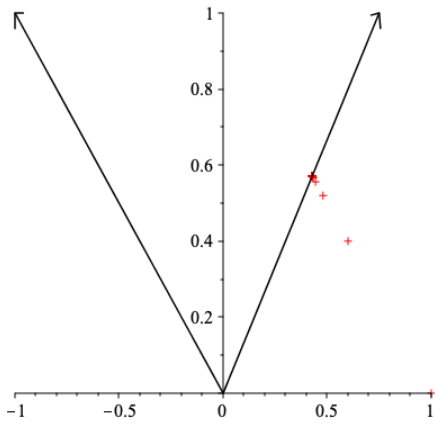
- The system tends to $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the long run
- The system stabilizes to $\begin{bmatrix} \frac{3}{7} \\ \frac{4}{7} \end{bmatrix}$ in the long run
- The systems keeps growing in the long run

Which [trajectory](#) graph represents the system? They all begin at (1,0) on the x -axis:

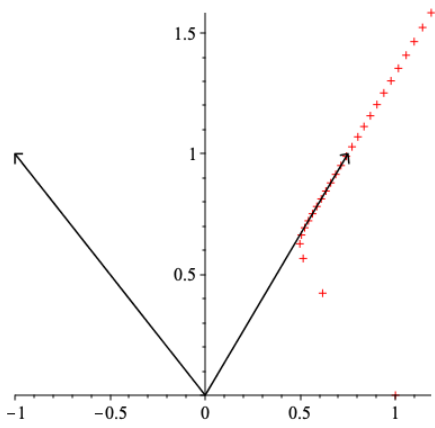
graph 1



graph 2



graph 3



- graph 1
- graph 2
- graph 3

Check

Question 11

Not complete

Points out of 1.00

To solidify and prepare for upcoming work, review and contemplate your knowledge and any questions that remain as related to definitions, concepts, computations, and examples from 5.6, including

- [eigenvector decomposition](#)
- [longterm behavior](#) or steady state
- geometry of [solutions](#) and a [trajectory](#) of the dynamical system
- relationship of the magnitude (absolute value) of the [eigenvalues](#) and the [longterm behavior](#), including the direction of greatest attraction or repulsion and which term is dominant in the [longterm](#) and what that means for [trajectories](#)

and consider 5.1 and 5.2, including

- [eigenvector](#), [eigenvalue](#), and [eigenspace](#)
- [eigenvectors](#) and difference equations on p. 279
- nonzero [eigenvalues](#) and [what makes a matrix invertible](#)
- [characteristic equation](#)
- similar matrices via a similarity transformation (compare with what we did in 2.7 and [computer graphics](#) to [rotate about a point other than the origin](#))
- application to dynamical systems

and 3.1, 3.2, 3.3, including

- [determinant](#) computations: [diagonal method](#) or [cofactor expansion](#) method ([Laplace expansion](#))
- [determinant](#) of a [triangular matrix](#) or a [transpose of a matrix](#)
- impact of [row operations](#) on [determinants](#)
- connection of [determinants](#) to [invertibility](#) and [what makes a matrix invertible](#)
- [determinants](#) as [area of parallelogram](#) or [volume of parallelepiped](#) and the impact of [row operations](#)

Since the material builds on itself, consider whether there is any material from before this module that you want to brush up on:

1.1

- algebra of linear equations: [coefficients](#) and variables
- geometry of linear equations in 2D and 3D: [lines](#) and [planes](#)
- [solution set](#): inconsistent: 0 [solutions](#); [consistent](#): 1 [unique](#) solution or [infinite solutions](#)
- matrix of a linear system: [coefficient](#) matrix, [augmented matrix](#), [triangular](#) form
- [row equivalent](#) systems
- algorithm for solving a linear system using [elementary row operations](#) of [replacement](#), [interchange](#), and [scaling](#)

1.2

- matrix of a linear system: [row echelon form](#) ([Gaussian](#)), [reduced row echelon form](#) ([Gauss-Jordan](#))
- [pivots](#): [pivot](#) position of a matrix, [pivot](#) column of a matrix
- row reduction algorithm we will most commonly use: [elimination](#) by forward phase and back substitution to [row echelon form](#)
- [solution set](#): inconsistent: 0 [solutions](#); [consistent](#): 1 [unique](#) solution or [infinite solutions](#) with [free variables](#) and [parametric solutions](#)

1.3

- algebra of [vectors](#): coordinates, [addition of vectors](#), [scalar multiplication of vectors](#), properties like [associativity](#) under addition (property ii on p. 29), a [linear combination](#) with [weights](#), zero [vector](#), [span](#) of a set of [vectors](#)=all the [linear combinations](#), is a [vector](#) in the [span](#)?, [vector](#) equation \rightarrow [augmented matrix](#)
- geometry of [vectors](#) in 2D and 3D: directed segment, [parallelogram](#) for addition, on same [line](#) for [scalar multiplication of vectors](#), origin=zero [vector](#), a [linear combination](#) geometrically in the [plane](#) or 3D, [span](#)=all the [linear combinations](#) geometrically in the [plane](#) or 3D, spaces of subsets of R^n [spanned](#) by [vectors](#)

1.4

- algebra of [matrix vector equation](#) $A\vec{x} = \vec{b}$:
 - [multiply a matrix and a column vector](#) by [linear combinations](#) of the columns of A using [weights](#) from \vec{x}
 - [span of the columns](#) of A = set of all [linear combinations](#) of the columns of A
 - [matrix vector equation](#) \rightarrow [vector](#) equation \rightarrow [augmented matrix](#)

- equations [generic vectors](#) \vec{b} must satisfy to be in the [span](#) (Example 3)
- [dot products](#) of rows of A with \vec{x} ,
- geometry of [solutions](#) of [matrix vector equation](#) $A\vec{x} = \vec{b}$: spaces of subsets of \mathbb{R}^3 [spanned](#) by the column [vectors](#) of A , geometry of such spaces (Figure 1)
- Theorem 4: relationship of [consistency](#) of $A\vec{x} = \vec{b}$ to always being a [linear combination](#) to [spanning](#) the entire \mathbb{R}^m , where m is the number of rows, to having a [pivot](#) position in every row of A .
- [identity matrix](#) I

1.5

- algebra of [homogeneous systems](#): $A\vec{x} = \vec{0}$
- algebraic [solutions](#) of homogenous systems always include the [trivial](#) solution= $\vec{0}$. nontrivial [solutions](#), if any exist, are [parameterized](#) in [parametric vector](#) form using [free variables](#) to express those as well as the variables with [pivots](#) and then decomposed algebraically to showcase the algebra and geometry giving $t\vec{v}$ or $s\vec{u} + t\vec{v}$ or similar, where each [free variable](#) is attached to a [vector](#).
- geometry of [solutions](#) of [homogeneous systems](#) are geometric spaces through the origin like [lines](#), [planes](#), or hyper[planes](#)
- algebra of nonhomogeneous systems: $A\vec{x} = \vec{b}$
- [solutions](#) of non[homogeneous system](#)s in [parametric vector](#) form can be decomposed algebraically to showcase the algebra and geometry like $\vec{p} + t\vec{v}$, [vectors](#) ending on the [line parallel](#) to \vec{v} or $\vec{p} + s\vec{u} + t\vec{v}$, [vectors](#) ending on the [plane](#) parallel to the one [spanned](#) by \vec{u}, \vec{v} ...
- geometry of [solutions](#) of non[homogeneous system](#)s are geometric spaces translated away from the origin via adding \vec{p}

1.7

- [linearly independent](#) set of [vectors](#) and connection to a [homogeneous equation](#) having only the [trivial](#) solution
- linearly dependent set of [vectors](#) and connection to nontrivial [solutions](#) existing and providing a dependence relation
- geometry of [linearly independent](#) set of 2 [vectors](#): independent directions in space versus along the same [line](#) (Figure 1)
- geometry of [linearly independent](#) set of 3 or more [vectors](#): no one [vector](#) is in the [span](#) of the rest, i.e. they are all needed to [span](#) the space versus redundancy in the geometric space they [span](#) in the sense that they aren't all needed to generate the same space under [linear combinations](#) (Figure 2)
- [linearly independent](#) columns of a matrix
- redundancy of $\vec{0}$ in a set of [vectors](#) $\{\vec{v}_1 = \vec{0}, \vec{v}_2, \dots, \vec{v}_n\}$ (Theorem 9)

1.8 and 1.9

- [linear transformation](#): addition and scalar multiplication
- left multiplication matrix representations
- [dilation](#), [projection](#), [reflection](#), [rotation](#), [shear](#) (see Examples 2-5 in 1.8, Examples 2-3 in 1.9, and tables 1-4 in 2.8)
- the algebraic image of the unit axes as a way to find the matrix of the transformation
- [range of a linear transformation](#): the algebraic or geometric images or outputs, e.g. of the unit square as a way to visualize the transformation and understand its effects
- the [range](#), image or output of a [sheared](#) shape

2.1

- matrices: [diagonal matrix](#) [and [main diagonal](#)], zero matrix
- matrix operations: [matrix addition](#), [scalar multiplication of a matrix](#), [matrix multiplication](#), powers of a matrix, left (or right) multiplication, [transpose of a matrix](#)
- [matrix multiplication](#) by [linear combinations](#) of the columns of A using [weights](#) from the corresponding column of B or by the [dot products](#) of a row of A with the corresponding column of B .
- algebraic properties that do hold for [matrix multiplication](#): [associativity](#) and one-sided distributivity
- algebraic properties that don't hold for [matrix multiplication](#): [commutativity](#)

2.2

- matrices: [invertible](#) ([nonsingular](#)) matrix, [noninvertible](#) ([singular](#)) matrix, [elementary matrix](#)
- [determinant and inverse of a 2x2 matrix](#)
- connection between [invertibility](#) and [unique solutions](#)
- [inverse](#) of a product of matrices and [inverse](#) of a [transpose](#)

2.3

- [what makes a matrix invertible](#) for a square matrix (Theorem 8 statements aside from f. and i., which we haven't covered)
- [condition number](#) (numerical note on p. 123)

2.7

- 2D and 3D [computer graphics](#) as columns of a matrix --- connect the dots!
- effects of 2D and 3D [linear transformations](#) from 1.8 and 1.9 and 3D transformations on figures--- algebra and matrix representation and geometry and visualization
- [homogeneous coordinates](#)
- [composite transformations](#) ABC is read right to left like functions, where C is the first action
- [rotate about a point other than the origin](#) (Figure 7)

2.8

- [subspace](#) properties: closed under addition and scalar multiplication
- spaces associated to a matrix: [column space](#) and [null space](#)
- [basis](#): [linearly independent spanning](#) set
- [basis](#) for [column space](#) as the [pivot](#) columns
- [basis](#) for [null space](#) as the [vectors](#) attached to [free variables](#) in [parametric solutions](#) of the [homogeneous system](#) $A\vec{x} = \vec{0}$

2.9

- [dimension](#) of a space
- [rank](#) of a matrix ([dimension](#) of [column space](#))
- [nullity](#) of a matrix ([dimension](#) of [null space](#))
- [rank nullity theorem](#) (Theorem 14)
- [what makes a matrix invertible](#) continued: adding [rank](#) and [nullity](#) to Theorem 8 when the matrix is square

6.1

- [inner product](#) of \vec{u} and \vec{v} and connection to the [dot product](#)
- [length](#) or [norm](#) of a [vector](#)
- [orthogonal vectors](#)

When you have finished reviewing and reflecting, select one of the following (both receive full credit)

- I currently have no questions
- I will continue solidifying and understand that help is available in Dr. Sarah's more extensive feedback that follows below each question after I finish and open back up an entire practice quiz (this is more extensive than the hints that I can access during the open quiz), in Dr. Sarah's glossary/Wiki which is embedded into ASUlearn from the linked terms, in Dr. Sarah's office hours and forum, and in Math Lab and Tutoring

Check

◀ [5.6 interactive video](#)

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[second chance 5.6 practice quiz](#) ▶