You can preview this quiz, but if this were a real attempt, you would be blocked because:

This quiz is not currently available

Question **1**

Not complete

Points out of 4.00

First compute the length/norm of $\vec{w} = \begin{bmatrix} 2 \\ -1 \\ -5 \end{bmatrix}$.

Then create a unit vector pointing in the same direction.

Compute the length/norm and write your response precisely in the following format to represent the square root of a simplified integer, **like sqrt(1)**, with no extra characters or spaces

$||\vec{w}|| = \sqrt{\vec{w} \cdot \vec{w}} =$

Next create a unit vector pointing in the same direction by dividing each coordinate of the vector by its norm. Write your responses like 2/sqrt(1), with no extra characters or spaces.

Check

Question **2**

Not complete

Points out of 2.00

Find a unit vector in the direction of the vector $\begin{bmatrix} -3 \\ 4 \end{bmatrix}$.

Type integers or simplified fractions with no extra spaces or characters, like -16/31

Check

Question **3**

Not complete

Points out of 4.00

Find the distance between $\vec{u} = \begin{bmatrix} 0 \\ -4 \\ 2 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} -4 \\ -1 \\ 9 \end{bmatrix}$

In 1.3 practice problems we saw that we could create the line connecting two vectors by making it parallel to $\vec{u} - \vec{v}$, the off-diagonal of the parallelogram they form.

We can use this reasoning here to find the distance between any two vectors, by finding the length of $||\vec{u} - \vec{v}||$

First, what is $\vec{u} - \vec{v}$? Simplify your response.

What is the length/norm of $\vec{u} - \vec{v}$? Write your response like sqrt(1) without any extra characters or spaces.

Check

Question **4**

Not complete

Points out of 2.00

Determine if the following vectors are orthogonal

$\vec{u} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$.

First, what is the dot product of the two vectors? Simplify your response.

[ ]

Are the vectors orthogonal?

○yes

○no

Check

Question **5**

Not complete

Points out of 2.00

Open
https://www.geogebra.org/m/apayedjs
which shows 2 vectors $\vec{u}$, $\vec{v}$, the coordinate axes in 3-space and the $xy0$-plane shaded.

Turn the graph so that you can see whether the vectors are orthogonal or not. What is the dot product of the 2 vectors?

○0

○nonzero

Next, create a "head on" view of the plane the 2 vectors are on by dragging the vectors on top of each other and having the other 2 axes overlap. In that visualization, are they the same length or does one stick out further?

○u and v are the same lengths

○u is longer

○v is longer

Check

Question **6**

Not complete

Points out of 1.00

Determine if the following vectors are orthogonal

$\vec{u} = \begin{bmatrix} 10 \\ 3 \\ -1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$.

Are the vectors orthogonal?

○yes

○no

Check

Question **7**

Not complete

Points out of 2.00

Open
https://www.geogebra.org/m/gctjpnys
from Juan Carlos Ponce Campuzano, University of Queensland, Australia, which we saw in 1.8 and 1.9. The vectors shown in the graph are the column vectors of the matrix, i.e. the image of the unit axes under the linear transformation.

Which examples have columns that are all mutually orthogonal? We can see this algebraically by performing dot products on each pairing from the columns, or we can see this geometrically by looking at the transformed cube and seeing whether the faces stay orthogonal to each other, via a rectangular box or cube as the output or image of the transformed figure

Choose all that have columns that are all mutually orthogonal. Choose all that apply!

☐Example 1

☐Example 2

☐Example 3

☐Example 4

☐Example 5

In Example 4, after you Apply T, turn the figure so that you can see the transformed cube on the top left of the screen. Does example 4 preserve the orthogonality of the cube?

○yes

○no, the cube has been distorted so that the faces are no longer orthogonal

Check

Question **8**

Not complete

Points out of 3.00

Suppose the vector $\vec{y}$ is orthogonal to both vectors $\vec{u}$ and $\vec{v}$. We'll show that $\vec{y}$ is orthogonal to every vector $\vec{w}$ in the Span $\{\vec{u}, \vec{v}\}$.

The span is the set of linear combinations of the vectors $c_1\vec{u} + c_2\vec{v}$.

So take the dot product: $\vec{y} \cdot (c_1\vec{u} + c_2\vec{v})$, and we'll use linearity to pair $\vec{y}$ with each of the vectors as follows:

Distribute $\vec{y}\cdot$ among the sum, by linearity (write this out on paper).

Now apply the linearity property of pulling out the scalars (write this out on paper).

Then apply the fact that $\vec{y}$ was orthogonal to $\vec{u}$ and $\vec{v}$.

What do we have after this last step?

c_1

[ ]

+ c_2

[ ]

=

[ ]

Check

Question **9**

Not complete

Points out of 2.00

Look at $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. What linear transformation is this?

○the identity acting on 2-space written in homogeneous coordinates

○projection onto the $\begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$ plane in 3-space

○other

The nullspace of A^T, the transpose, is always orthogonal to the column space of A. Notice that in this example, $A^T = A$ but that usually won't be the case. To see the orthogonality, open
https://www.geogebra.org/m/qxgg7vrm
and move the sliders. What do we see?

○the nullspace is the z-axis, which is orthogonal to the column space, the $\begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$ plane

○the nullspace is the origin and the column space is the entire space

○other

Check

Question **10**

Not complete

Points out of 4.00

Least squares analysis is very useful in many real-life applications where we want a best fit line (it's linear!). However, typically, no line exactly goes through all of the given points. We want to find the line that fits as closely as possible to all of the points (we minimize the squared vertical distance between the points and the line using the Pythagorean theorem).

Typically the matrix we are looking at isn't square, so we can't take its inverse, but we can take the inverse of $(A^T A)$! Then the key to finding the y-intercept and slope $\begin{bmatrix} b \\ m \end{bmatrix} = (A^T A)^{-1} A^T \vec{y}$ comes from a property of subspaces: The null space of A^T, the transpose of A is orthogonal to the column space of A.

Let's look at an example of $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \xrightarrow{r_2' = -r_1 + r_2} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 3 \end{bmatrix} \xrightarrow{r_3' = -r_1 + r_3} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 2 \end{bmatrix} \xrightarrow{r_3' = -2r_2 + r_3} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

So both column 1 and column 2 are pivot columns and $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$ is a basis for the column space of A

To find the null space of A^T, notice that $A^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ and we want to solve for solutions of $A^T \vec{x} = \vec{0}$ and write a basis. The augmented matrix is $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 \end{bmatrix}$ which reduces to $\xrightarrow{r_2' = -r_1 + r_2} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$

Here x_3 is free so set it to t.

Use row 2 to solve for $x_2 = -2t$.

Use row 1 to solve for $x_1 = -x_2 - x_3 = -(-2t) - t = t$.

Thus the nullspace is $t \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ and $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}$ is a basis for that line.

What is $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$?

[ ]

What is $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$?

[ ]

This shows that the basis vector we found for the nullspace of A^T is

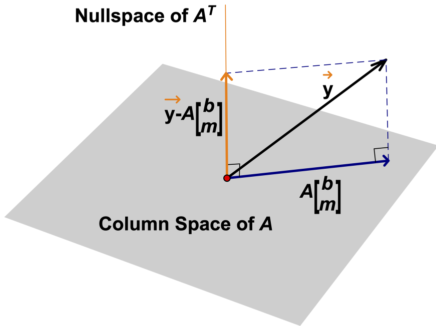
- ○ parallel
- ○ orthogonal

to both basis vectors we found for the column space of A.

Since the basis vectors generate the entire spaces, any vector in the null space of A^T must be

- ○ parallel
- ○ orthogonal

to any vector in the column space of A.



We can see that $A\begin{bmatrix} b \\ m \end{bmatrix}$ is the vector in the column space of A that is closest to $\vec{y}$, the projection of $\vec{y}$ onto the plane. In addition, $\vec{y} - A\begin{bmatrix} b \\ m \end{bmatrix}$ is in the null space of A^T, which gives us the way to compute best fit lines! Lots of course topics connect: column space, inverse, matrix algebra, matrix multiplication, nullspace, orthogonality, projection, solving linear equations, and transpose.

Check

Question **11**

Not complete

Points out of 1.00

To solidify and prepare for upcoming work, review and contemplate your knowledge and any questions that remain as related to definitions, concepts, computations, and examples from 6.1, including

- inner product of $\vec{u}$ and $\vec{v}$ and connection to the dot product
- length or norm of a vector
- orthogonal vectors

and consider 1.8 and 1.9, including

- linear transformation: addition and scalar multiplication
- left multiplication matrix representations
- dilation, projection, reflection, rotation, shear (see Examples 2-5 in 1.8, Examples 2-3 in 1.9, and tables 1-4 in 2.8)
- the algebraic image of the unit axes as a way to find the matrix of the transformation
- range of a linear transformation: the algebraic or geometric images or outputs, e.g. of the unit square as a way to visualize the transformation and understand its effects
- the range, image or output of a sheared sheep

Since the material builds on itself, consider whether there is any material from before this module that you want to brush up on:

1.1

- algebra of linear equations: coefficients and variables
- geometry of linear equations in 2D and 3D: lines and planes
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions
- matrix of a linear system: coefficient matrix, augmented matrix, triangular form
- row equivalent systems
- algorithm for solving a linear system using elementary row operations of replacement, interchange, and scaling

1.2

- matrix of a linear system: row echelon form (Gaussian), reduced row echelon form (Gauss-Jordan)
- pivots: pivot position of a matrix, pivot column of a matrix
- row reduction algorithm we will most commonly use: elimination by forward phase and back substitution to row echelon form
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions with free variables and parametric solutions

1.3

- algebra of vectors: coordinates, addition of vectors, scalar multiplication of vectors, properties like associativity under addition (property ii on p. 29), a linear combination with weights, zero vector, span of a set of vectors=all the linear combinations, is a vector in the span?, vector equation $\longrightarrow$ augmented matrix
- geometry of vectors in 2D and 3D: directed segment, parallelogram for addition, on same line for scalar multiplication of vectors, origin=zero vector, a linear combination geometrically in the plane or 3D, span=all the linear combinations geometrically in the plane or 3D, spaces of subsets of R^n spanned by vectors

1.4

- algebra of matrix vector equation $A\vec{x} = \vec{b}$:
  - multiply a matrix and a column vector by linear combinations of the columns of *A* using weights from $\vec{x}$
  - span of the columns of *A* = set of all linear combinations of the columns of *A*
  - matrix vector equation $\longrightarrow$ vector equation $\longrightarrow$ augmented matrix
  - equations generic vectors $\vec{b}$ must satisfy to be in the span (Example 3)
  - dot products of rows of *A* with $\vec{x}$,
- geometry of solutions of matrix vector equation $A\vec{x} = \vec{b}$: spaces of subsets of R^3 spanned by the column vectors of A, geometry of such spaces (Figure 1)
- Theorem 4: relationship of consistency of $A\vec{x} = \vec{b}$ to always being a linear combination to spanning the entire R^m, where *m* is the number of rows, to having a pivot position in every row of *A*.
- identity matrix *I*

1.5

- algebra of homogeneous systems: $A\vec{x} = \vec{0}$
- algebraic solutions of homogenous systems always include the trivial solution=$\vec{0}$. nontrivial solutions, if any exist, are parameterized in parametric vector form using free variables to express those as well as the variables with pivots and then decomposed algebraically to showcase the algebra and geometry giving $t\vec{v}$ or $s\vec{u} + t\vec{v}$ or similar, where each free variable is attached to a vector.
- geometry of solutions of homogeneous systems are geometric spaces through the origin like lines, planes, or hyperplanes
- algebra of nonhomogeneous systens: $A\vec{x} = \vec{b}$
- solutions of nonhomogeneous systems in parametric vector form can be decomposed algebraically to showcase the algebra and geometry like $\vec{p} + t\vec{v}$, vectors ending on the line parallel to $\vec{v}$ or $\vec{p} + s\vec{u} + t\vec{v}$, vectors ending on the plane parallel to the one spanned by $\vec{u}, \vec{v}$...
- geometry of solutions of nonhomogeneous systems are geometric spaces translated away from the origin via adding $\vec{p}$

1.7

- linearly independent set of vectors and connection to a homogeneous equation having only the trivial solution
- linearly dependent set of vectors and connection to nontrivial solutions existing and providing a dependance relation
- geometry of linearly independent set of 2 vectors: independent directions in space versus along the same line (Figure 1)
- geometry of linearly independent set of 3 or more vectors: no one vector is in the span of the rest, i.e. they are all needed to span the space versus redundancy in the geometric space they span in the sense that they aren't all needed to generate the same space under linear combinations (Figure 2)
- linearly independent columns of a matrix
- redundancy of $\vec{0}$ in a set of vectors $\{\vec{v}_1 = \vec{0}, \vec{v}_2, \ldots \vec{v}_n\}$ (Theorem 9)

2.1

- matrices: diagonal matrix [and main diagonal], zero matrix
- matrix operations: matrix addition, scalar multiplication of a matrix, matrix multiplication, powers of a matrix, left (or right) multiplication, transpose of a matrix
- matrix multiplication by linear combinations of the columns of A using weights from the corresponding column of B or by the dot products of a row of A with the corresponding column of B.
- algebraic properties that do hold for matrix multiplication: associativity and one-sided distributivity
- algebraic properties that don't hold for matrix multiplication: commutativity

2.2

- matrices: invertible (nonsingular) matrix, noninvertible (singular) matrix, elementary matrix
- determinant and inverse of a 2x2 matrix
- connection between invertibility and unique solutions
- inverse of a product of matrices and inverse of a transpose

2.3

- what makes a matrix invertible for a square matrix (Theorem 8 statements aside from f. and i., which we haven't covered)
- condition number (numerical note on p. 123)

2.8

- subspace properties: closed under addition and scalar multiplication
- spaces associated to a matrix: column space and null space
- basis: linearly independent spanning set
- basis for column space as the pivot columns
- basis for null space as the vectors attached to free variables in parametric solutions of the homogeneous system $A\vec{x} = \vec{0}$

2.9

- dimension of a space
- rank of a matrix (dimension of column space)
- nullity of a matrix (dimension of null space)
- rank nullity theorem (Theorem 14)
- what makes a matrix invertible continued: adding rank and nullity to Theorem 8 when the matrix is square

When you have finished reviewing and reflecting, select one of the following (both receive full credit)

○ I currently have no questions

○ I will continue solidifying and understand that help is available in Dr. Sarah's more extensive feedback that follows below each question after I finish and open back up an entire practice quiz (this is more extensive than the hints that I can access during the open quiz), in Dr. Sarah's glossary/Wiki which is embedded into ASULearn from the linked terms, in Dr. Sarah's office hours and forum, and in Math Lab and Tutoring

Check