

1. To reduce 1.1#26 (Problem 2 on PS1) to Gaussian by-hand we can apply

$\begin{bmatrix} 2 & 4 & f \\ c & d & g \end{bmatrix} \xrightarrow{r'_2 = -\frac{c}{2}r_1 + r_2} \begin{bmatrix} 2 & 4 & f \\ 0 & -2c + d & -\frac{c}{2}f + g \end{bmatrix}$ . The book asks us what conditions we have on  $c$  and  $d$  to ensure consistency no matter what values  $f$  and  $g$  take on. The reasoning is:

- a) The system is always consistent because Maple shows us that we have full pivots via `ReducedRowEchelon` equaling  $\begin{bmatrix} 1 & 0 & -\frac{1}{2}\frac{df-4g}{-d+2c} \\ 0 & 1 & -\frac{1}{2}\frac{cf-2g}{-d+2c} \end{bmatrix}$ , so  $c$  and  $d$  can take on any values, no matter what  $f$  and  $g$  are.
- b) The system is never consistent because we always obtain  $[0 \ 0 \ \text{nonzero}]$ , so there are no values of  $c$  and  $d$  that work.
- c) We must have  $d \neq 2c$  because otherwise we'll obtain  $[0 \ 0 \ \text{nonzero}]$  whenever  $g \neq \frac{c}{2}f$ .
- d) Either  $d \neq 2c$  to give us a pivot for  $x_2$  or we make  $x_2$  free by setting  $g = \frac{c}{2}f$  whenever  $d = 2c$ .
- e) Other

Solutions

1. c)