

For $A_{m \times n}$ the following are equivalent (TFAE):

- a. For each \vec{b} in \mathbb{R}^m , $A\vec{x} = \vec{b}$ has at least one solution (one or more \vec{x} in \mathbb{R}^n)
- b. Each \vec{b} in \mathbb{R}^m is a linear combination of the columns of A
- c. The columns of A span \mathbb{R}^m
- d. A has a pivot position in every row

For $A_{2 \times 3}$: $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ has the augmented matrix $\left[A \begin{array}{c} b_1 \\ b_2 \end{array} \right]$

Compare A as $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ with A as $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$