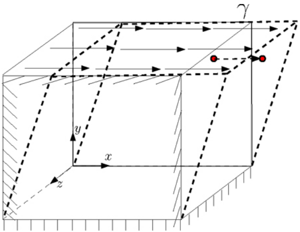
Connection of Matrix Operations and Determinants

Compute the matrices and their determinants and compare. Does the determinant change? If so, how is it related to the original? Also, what kind of linear transformations are the first three?

1. $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ versus determinant of matrix after $r_2' = -3r_1 + r_2$
2. $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ versus determinant of matrix after $r_1 \leftrightarrow r_2$
3. $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ versus determinant of the matrix after $r_2' = cr_2, c \neq 0$
4. $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ versus determinant of the transpose of the matrix



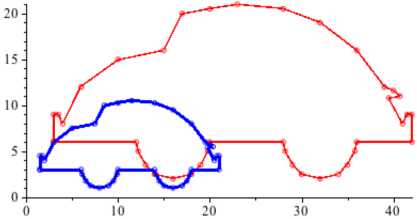


Image 1: Modeling of Hot-Mix Asphalt Compaction: A Thermodynamics-Based Compressible Viscoelastic Model [FHWA-HRT-10-065], rest of images made using VLA program by Herman and Pepe *Visual Linear Algebra*

- $r_j' = cr_i + r_j$ shear $\begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$.object same determinant
- $r_i \leftrightarrow r_j$ reflect $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.object negative determinant
- $r_j' = cr_j$ scale $\begin{bmatrix} c & 0 \\ 0 & 1 \end{bmatrix}$.object scales determinant
- transpose preserves determinant

Determinant of Triangular and Inverses

A triangular has 0s below the diagonal (such as in Gaussian form), or 0s above the diagonal:

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 8 & 9 \\ 0 & 0 & 0 & 10 \end{vmatrix} \text{ or } \begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & 5 & 0 & 0 \\ 3 & 6 & 8 & 0 \\ 4 & 7 & 9 & 10 \end{vmatrix}$$

What is the determinant of a triangular matrix?

What is the determinant of the inverse of a matrix?

- write the inverse of $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$. What is its determinant and how does it compare to the original?
- via matrix algebra
- via elementary row operations

$\begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix}$ det$A$ = detA^T so examine the rows and the *unit span*

$tr_1 + sr_2$, $0 \leq s, t \leq 1$

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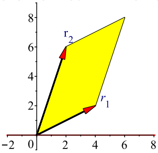

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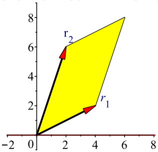

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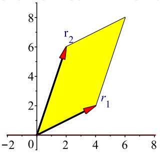

$tr_1 + sr_2,\ 0 \leq s, t \leq 1$

strict Gaussian $r_2' = -\frac{1}{2} r_1 + r_2$ or equivalently the shear

$\begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 0 & 5 \end{bmatrix}$ preserves determinant

In general $\begin{bmatrix} 1 & 0 \\ t & 1 \end{bmatrix}$ or $r_2' = tr_1 + r_2$ takes the second row to a vector that ends on the line parallel to _ through the tip of _

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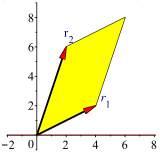

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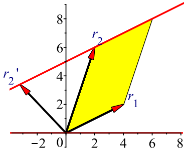

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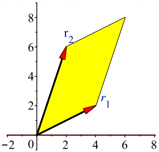

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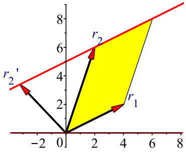
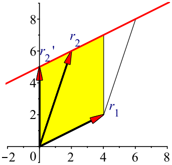
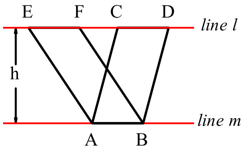


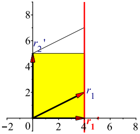




Note: since we are acting on the rows rather than the columns it isn't visualized as a vertical shear—it is an r_1 shear

strict Gauss-Jordan $r_1' = -\frac{2}{5}r_2 + r_1$ or equivalently

$$\begin{bmatrix} 1 & -\frac{2}{5} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}$$



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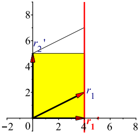

determinant = area = 20

Strict replacements shears unit span parallelograms to rectangles with the same area. We may have had to swap rows to make this work, changing only the sign of the determinant.

|determinant| = area for 2 column vectors in a 2×2 matrix

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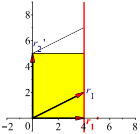
$\begin{bmatrix} 1 & -\frac{2}{5} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}$ 

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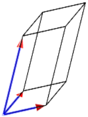

volume of unit span parallelepiped

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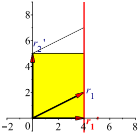
$\begin{bmatrix} 1 & -\frac{2}{5} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}$ 

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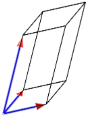 volume of unit span parallelepiped

0 determinant?

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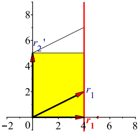

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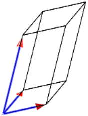
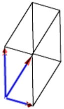


volume of unit span parallelepiped

0 determinant? degenerate figure—smushed—like 3 vectors all in the same plane giving 0 volume



span is not all of $\mathbb{R}^3$

all images except first on page 1 made using VLA program by Herman and Pepe *Visual Linear Algebra*