Linear Algebra: Sample Test 1 Questions

Part 1: Fill in the Blank Questions (30 points) There may be more than one possible answer for a fill-in-the-blank question. Full credit answers are ones that demonstrate deep understanding of linear algebra from class and homework.

1. In linear algebra, a vector means _____

2. An augmented matrix corresponding to three equations reduces to

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

   The pivots are _____

3. What are the solution(s), if any, in #2? _____

4. Multiply $\begin{bmatrix} 5 & 8 \\ -2 & 3 \end{bmatrix}$ by-hand via $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ (show work, but no need to reduce) _____

5. Adding two vectors $\vec{v}_1$ and $\vec{v}_2$ gives _____

6. The row operation which turns $\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 4 & -1 & 3 & -6 \end{bmatrix}$ to $\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 0 & 7 & -1 & -6 \end{bmatrix}$ is (like $r_3' = -5r_1 + r_3$)

   _____

7. If I use the implicitplot3d command in Maple on the equations corresponding to the rows of the augmented matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 8 & 9 \end{bmatrix}$ we would see _____ intersecting in _____

8. We have repeatedly seen that we must be careful with Maple's linear algebra commands, because we can sometimes get incorrect answers. An example is when:

   _____

9. In problem set 2, the center of gravity was an example of the linear algebra concept _____

10. If A is an $n \times n$ matrix, and $\vec{x}$ and $\vec{b}$ are $1 \times n$ vectors, then $A\vec{x} = \vec{b}$ has _____ solution(s).

11. $\begin{bmatrix} -1 & 3 \\ 2 & 4 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix}$ = (show work, but no need to reduce) _____

12. The inverse of $\begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix}$ is (show work, but no need to reduce) _____

13. To solve $A\vec{x} = \vec{b}$ with A as in the last question, we can _____

14. If the condition number is on the order of 10^4 then that tells us that we may lose up to _____

15. In linear algebra, span means _____

16. $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$ has columns that _____

17. A real-life application of [a topic] is _____

18. If A is an invertible $n \times n$ matrix, and $\vec{x}$ and $\vec{b}$ are $n \times 1$ vectors, then $A\vec{x} = \vec{b}$ has _____ solution(s).

Part 2: Computations and Interpretations (40 points)

There will be some by-hand computations and interpretations, like those you have had previously for homework, clicker questions and in the problem sets. You are not expected to remember page numbers or Theorem numbers, but you are expected to be comfortable with definitions, "big picture" ideas, computations, analyses...

You can expect this section to be a question with numerous parts, adapted from (or combining) these types of questions (most have solutions on ASULearn) :

- by-hand Gaussian of matrices and connections:

  1.2 #19
  Problem Set 1 #1 or #2

- and/or the algebra and geometry of vectors and connections:

  1.3 #15
  1.4 #13
  1.7 #9
  Problem Set 2 #2 or #3
  worksheet extension of 1.4 #33

- and/or the algebra of matrices and connections:

  2.1 #9, 21, 23
  Clicker in 2.1 and 2.2 #7
  2.2 #13, 17, 21, 23
  2.3 #19, 21, 23
  Problem Set 3 #1, #2 or #4

Part 3: True/False (30 points) Follow the directions below each. One section is Circle True OR correct the statements as directed, and the other is Circle True OR provide a counterexample.

Circle True OR correct the statements as directed:

a) The solution set of a linear system involving variables $x_1, ..., x_n$ is all lists of numbers $(s_1, ..., s_n)$ that makes each equation in the system a true statement when the values $s_1, ..., s_n$ are substituted for $x_1 ... x_n$, respectively.
Circle True OR (only if false) correct the statement after is.

b) $\begin{bmatrix} 1 & 4 & -2 \\ 0 & -12+h & 0 \end{bmatrix}$ is consistent as long as h is not 12
Circle True OR (only if false) correct the statement after is consistent

c) The vector equation $x_1 \begin{bmatrix} 5 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$ is equivalent to the matrix equation $\begin{bmatrix} 5 & 1 & -3 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$

Circle True OR (only if false) correct the statement after equation.

d) The plane spanned by $\begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$ includes many vectors in that plane that are not on the same lines as the spanning vectors, such as $\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$
Circle True OR (only if false) correct the statement after such as

e) Two vectors that are linearly independent in $\mathbb{R}^2$ are $S = \{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}\}$
Circle True OR (only if false) correct the statement after are.

f) The equation $\vec{x} = \vec{p} + t\vec{v}$ describes a line through $\vec{p}$ parallel to $\vec{v}$
Circle True OR (only if false) correct the statement after describes.

g) If $A = \begin{bmatrix} 4 & 6 \\ 20 & 7 \end{bmatrix}$ then $5A = \begin{bmatrix} 20 & 6 \\ 20 & 7 \end{bmatrix}$
Circle True OR (only if false) correct the statement after $5A =$

h) Each column of AB is a linear combination of the columns of B using weights from the corresponding columns of A.
Circle True OR (only if false) correct the statement after of

i) The transpose of a product of matrices equals the product of their transposes in the same order
Circle True OR (only if false) correct the statement after the

j) $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} .A$ is the same as modifying A via $r_2' = 3r_1 + r_2$
Circle True OR (only if false) correct the statement after via

k) If the equation $A\vec{x} = \vec{0}$ has a nontrivial solution, then $A_{n\times n}$ has fewer than n pivot positions.
Circle True OR (only if false) correct the statement after then

Circle True OR provide a counterexample:

l) If one row in an echelon (Gaussian) form of an augmented matrix is [0 0 0 5 0] then the associated linear system is inconsistent.
Circle True OR provide a counterexample

m) Any system of 3 linear equations in 2 unknowns is always inconsistent
Circle True OR provide a counterexample

n) A not square can never have only the trivial solution for $A\vec{x} = \vec{0}$.
Circle True OR provide a counterexample

o) If A is an nxn matrix then the equation $A\vec{x} = \vec{b}$ has at least one solution for each $\vec{b}$ in $\mathbb{R}^n$.
Circle True OR provide a counterexample