Linear Algebra: Sample Test 1 Questions

Part 1: Fill in the Blank Questions (30 points)

There may be more than one possible answer for a fill-in-the-blank question. Full credit answers are ones that demonstrate deep understanding of linear algebra from class and homework.

1. In linear algebra, a vector means an ordered column representing magnitude and direction

   From 1.3 and the ASULearn glossary

2. An augmented matrix corresponding to three equations reduces to $\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ The pivots are all three 1s

   Adapted from 1.2 Problem 3

3. What are the solution(s), if any, in #2? no solutions because 0x+0y=1 by row 3

   Adapted from 1.1 and 1.2 hw and clicker questions

4. Multiply $\begin{bmatrix} 5 & 8 \\ -2 & 3 \end{bmatrix}$ by-hand via $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ (show work, but no need to reduce) $\begin{bmatrix} 5 \times -1 + 8 \times 1 \\ -2 \times -1 + 3 \times 1 \end{bmatrix}$ - can use the dot product method or the linear combination method.

   Adapted from 1.4 Problem 3

5. Adding two vectors $\vec{v}_1$ and $\vec{v}_2$ gives the diagonal of the parallelogram

   Adapted from 1.3 Problem 1

6. The row operation which turns $\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 4 & -1 & 3 & -6 \end{bmatrix}$ to $\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 0 & 7 & -1 & -6 \end{bmatrix}$ is (like $r_3' = -5r_1 + r_3$)

   $r_3' = -4r_1 + r_3$

   See 1.1 #31

7. If I use the implicitplot3d command in Maple on the equations corresponding to the rows of the augmented matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 8 & 9 \end{bmatrix}$ we would see 3 planes intersecting in a point

   Adapted from Problem Set 1 # 1

8. We have repeatedly seen that we must be careful with Maple's linear algebra commands, because we can sometimes get incorrect answers. An example is when:

   we use decimals instead of fractions (or) we use ReducedRowEchelon on a matrix with unknowns in the array

   Adapted from Coffee mixing (or) Problem Set 1 hint sheet

9. In problem set 2, the center of gravity was an example of the linear algebra concept linear combination of vectors

   See problem Set 2 #1

10. If A is an $n \times n$ matrix, and $\vec{x}$ and $\vec{b}$ are $1 \times n$ vectors, then $A\vec{x} = \vec{b}$ has no solution(s). Adapted from a combination of Clicker questions 1.1 and 1.2 #4 and 1.4 classroom notes - it is 0 because it should be $n \times 1$ vectors to give 0, 1, or infinite solutions

11. $\begin{bmatrix} -1 & 3 \\ 2 & 4 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix}$ = (show work, but no need to reduce) $\begin{bmatrix} -1\times 4+3\times -2 & -1\times -2+3\times 3 \\ 2\times 4+4\times -2 & 2\times -2+4\times 3 \\ 5\times 4-3\times -2 & 5\times -2+-3\times 3 \end{bmatrix}$

    See 2.1 number 5

12. The inverse of $\begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix}$ is (show work, but no need to reduce) $\frac{1}{4\cdot 3-(-2)(-2)} \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$

    See 2.2 number 1

13. To solve $A\vec{x} = \vec{b}$ with A as in the last question, we can solve $\vec{x} = A^{-1}\vec{b}$ (or reduce $[A|b]$)

    See 2.2 number 5

14. If the condition number is on the order of 10^4 then that tells us that we may lose up to 4 digits of accuracy

    Problem Set 3 #3 (it measures the asymptotically worst case of how much the solutions to a system of equations can change with small variations in the system.)

15. In linear algebra, span means set of all linear combinations

    From 1.3 and the ASULearn glossary (can be 1 or infinite solutions for any $\vec{b}$ in the span)

16. $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$ has columns that are not l.i. (or do not span or are missing a pivot)

17. A real-life application of [a topic] is

    Hill cipher, computer efficiency and reliability... (answer would depend on the topic chosen)

18. If A is an invertible $n \times n$ matrix, and $\vec{x}$ and $\vec{b}$ are $n \times 1$ vectors, then $A\vec{x} = \vec{b}$ has 1 unique $\vec{x} = A^{-1}\vec{b}$ solution(s). Adapted from a combination of Clicker questions in 2.2 #2 and Clicker questions in 2.1 #5

Part 2: Computations and Interpretations (40 points)

There will be some by-hand computations and interpretations, like those you have had previously for homework, clicker questions and in the problem sets. You are not expected to remember page numbers or Theorem numbers, but you are expected to be comfortable with definitions, "big picture" ideas, computations, analyses...

You can expect this section to be a question with numerous parts, adapted from (or combining) these types of questions. See solutions on ASULearn and be sure you could do similar problems

- by-hand Gaussian of matrices and connections:

  1.2 #19
  Problem Set 1 #1 or #2

- and/or the algebra and geometry of vectors and connections:

  1.3 #15
  1.4 #13
  1.7 #9
  Problem Set 2#2 or #3
  worksheet extension of 1.4 #33

- and/or the algebra of matrices and connections:

  2.1 #9, 21, 23
  Clicker in 2.1 and 2.2 #7
  2.2 #13, 17, 21, 23
  2.3 #19, 21, 23
  Problem Set 3 #1, #2 or #4

Part 3: True/False (30 points) Follow the directions below each. One section is Circle True OR correct the statements as directed, and the other is Circle True OR provide a counterexample.

Circle True OR correct the statements as directed:

a) The solution set of a linear system involving variables $x_1, ..., x_n$ is all lists of numbers $(s_1, ..., s_n)$ that makes each equation in the system a true statement when the values $s_1, ..., s_n$ are substituted for $x_1 ... x_n$, respectively.

Circle (True) OR (only if false) correct the statement after is. True - Adapted from 1.1 #23 to correct it

b) $\begin{bmatrix} 1 & 4 & -2 \\ 0 & -12+h & 0 \end{bmatrix}$ is consistent ~~as long as h is not 12~~ for all h
Circle True OR (only if false) correct the statement after is consistent False - Adapted from 1.1 #21

c) The vector equation $x_1 \begin{bmatrix} 5 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$ is equivalent to the matrix equation

$\begin{bmatrix} 5 & 1 & -3 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$
Circle True OR (only if false) correct the statement after equation. False - Adapted from 1.4 #9

d) The plane spanned by $\begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$ includes many vectors in that plane that are not on the same lines as the spanning vectors, such as $\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$

Circle (True) OR (only if false) correct the statement after such as.

This is the example I told you to memorize from class.

e) Two vectors that are linearly independent in $\mathbb{R}^2$ are ~~$S = \{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}\}$~~ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

False: Adapted from clicker question 1.3 # 2 and Problem Set 2 # 2.

f) The equation $\vec{x} = \vec{p} + t\vec{v}$ describes a line through $\vec{p}$ parallel to $\vec{v}$

Circle (True) OR (only if false) correct the statement after describes. Adapted from 1.5 #23 d to correct it

g) If $A = \begin{bmatrix} 4 & 6 \\ 20 & 7 \end{bmatrix}$ then $5A =$ ~~$\begin{bmatrix} 20 & 6 \\ 20 & 7 \end{bmatrix}$~~ $\begin{bmatrix} 20 & 30 \\ 100 & 35 \end{bmatrix}$
Circle True OR (only if false) correct the statement after $5A =$

Clicker questions in 2.1 #3

h) Each column of AB is a linear combination of the columns of ~~B~~ A using weights from the corresponding columns of ~~A~~ B.
Circle True OR (only if false) correct the statement after of

2.1 #15 b)

i) The transpose of a product of matrices equals the product of their transposes in the ~~same order~~ reverse order
Circle True OR (only if false) correct the statement after the
2.1 #15 e)

j) $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$ is the same as modifying A via $r_2' = 3r_1 + r_2$

Circle (True) OR (only if false) correct the statement after via.
Clicker questions in 2.1

k) If the equation $A\vec{x} = \vec{0}$ has a nontrivial solution, then $A_{n\times n}$ has fewer than n pivot positions.

Circle (True) OR (only if false) correct the statement after then. 2.3 #11d)

Circle True OR provide a counterexample:

l) If one row in an echelon (Gaussian) form of an augmented matrix is [0 0 0 5 0] then the associated linear system is inconsistent.
Circle True OR provide a counterexample
$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \end{bmatrix}$
is a counterexample because this system has a unique solution
1.2 Problem 21 part e)

m) Any system of 3 linear equations in 2 unknowns is always inconsistent
Circle True OR provide a counterexample
$x + y = 0$
$x + 2y = 0$
$x + 3y = 0$
is a counterexample because $x = 0, y = 0$ is a solution
1.2 Problem 31

n) A not square can never have only the trivial solution.
Circle True OR provide a counterexample $A = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Clicker questions in 2.3 and Hill Cipher #2. False with l.i. columns.

o) If A is an nxn matrix then the equation $A\vec{x} = \vec{b}$ has at least one solution for each $\vec{b}$ in $\mathbb{R}^n$.
Circle True OR provide a counterexample 2.3 #11c). False if A not invertible, with the appropriate $\vec{b}$ outside the span of the columns of A, like $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ & $\vec{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$