Linear Algebra: Sample Test 3 Questions

Part 1: Fill in the Blank Questions (3 points each - 30 points total) There may be more than one possible answer for a fill-in-the-blank question. Full credit answers are ones that demonstrate deep understanding of linear algebra from class and homework.

1. A rotation that rotates counterclockwise by θ is represented in matrix form as

   $\underline{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}$ Adapted from 1.8, 1.9 and 2.7

2. The determinant of $\begin{bmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ by-hand gives (show work, but no need to reduce) $3(-1)^2 \begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} + 0(-1)^3 \begin{vmatrix} 2 & 2 \\ 0 & 2 \end{vmatrix} + 4(-1)^4 \begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix}$ See 3.1 number 1 and 15

3. A shear matrix is useful for turning a parallelogram into a rectangle OR geology OR cartoony animations that stretch figures OR representing replacement row operations as matrix multiplications...

   See Clicker questions in chapter 3 for example

4. An elementary matrix that represents a shear matrix is $\begin{bmatrix} 1 & 0 \\ -k & 1 \end{bmatrix}$ See 3.1 # 21 and 25, for example

5. An eigenvector turns matrix multiplication into scalar multiplication OR stays on the same line through the origin it started on OR is a $\vec{x}$ that satisfies $A\vec{x} = \lambda\vec{x}$

   See clicker questions in 5.1 for example.

6. $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ has <u>no</u> real eigenvalues for most θ

   See Problem Set 3 #4 for example.

7. A matrix that has all of $\mathbb{R}^2$ as its eigenspace is $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ OR $\begin{bmatrix} \cos\pi & -\sin\pi \\ \sin\pi & \cos\pi \end{bmatrix}$

   Problem Set 3 #4 or Eigenvector Clicker review questions for example.

8. If I use the implicitplot3d command in Maple on the equations corresponding to the rows of the augmented matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 5 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ we would see that the nullspace is a <u>line</u>

   Adapted from Problem Set 1 # 1 and 2.8#23 and Problem Set 4 #2

9. A basis for the column space of $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$ is $\underline{\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \right\}}$

   Adapted from class notes in 2.8, and Clicker questions in 2.8

10. If A is an $n \times n$ matrix with a zero determinant, and $\vec{x}$ and $\vec{b}$ are $1 \times n$ vectors, then $A\vec{x} = \vec{0}$ has <u>no</u> solution(s). Adapted from Clicker questions in chapter 3 - it is 0 because it should be $n \times 1$ vectors to give infinite solutions

11. If A is an $n \times n$ matrix with a non-zero determinant, and $\vec{x}$ and $\vec{b}$ are $1 \times n$ vectors, then $A\vec{x} = \vec{b}$ has <u>no</u> solution(s). Adapted from a combination of previous clicker questions it is 0 because it should be $n \times 1$ vectors to give 1 unique solution

Part 2: Computations and Interpretations (40 points)

There will be some by-hand computations and interpretations, like those you have had previously for homework, clicker questions and in the problem sets. You are not expected to remember page numbers or Theorem numbers, but you are expected to be comfortable with definitions, "big picture" ideas, computations, analyses...

You can expect this section to be a question with numerous parts, adapted from (or combining) these questions. See solutions on ASULearn/notes and be sure you could do similar problems:

2.7 #9
Clickers in 2.7 #4, 7, 8, 9
6.1 #15
3.1 #1
3.2 #42
3.3 #19, 25
2.8 #23
5.1 #2, 31
5.6 #3
Problem Set 4 #2, 3 or 4

Part 3: True/False (3.75 points each - 30 points total) Follow the directions below each:

Circle True OR correct the statements as directed:

a) To keep a car on a curved race track, we can perform the appropriate matrix operations in the following order ~~(Rotate).(Translate_to_curve).car~~ (Translate_to_curve).(Rotate).car
Circle True OR (only if false) correct the statement after order False - Clicker questions in 2.7 #7

b) det AB $\underline{=}$ detA det B

Circle True OR (only if false) correct the statement after $\underline{=}$ [True: 3.2 #37 and class notes]

c) The volume of the parallelopiped formed by the column vectors of a matrix that is not invertible is 0.

Circle True OR (only if false) correct the statement after is [True: 3.3 #25]

d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & k & 1 \end{bmatrix}$ is ~~not~~ invertible
Circle True OR (only if false) correct the statement after is [False: 3.1 # 21 and 25]

e) The column space of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ is a subspace of $\mathbb{R}^3$

Circle True OR (only if false) correct the statement after of [True: Adapted from 2.8 #11 and 13 and clicker questions in 2.8]

f) If the equation $A\vec{x} = \vec{0}$ has a nontrivial solution, then the nullspace of A is at least a line

Circle True OR (only if false) correct the statement after then. [True: Adapted from 2.8 #21c]

g) To find the eigenvalues of A, solve ~~by reducing A to echelon form~~ determinant$(A - \lambda I) = 0$
Circle True OR (only if false) correct the statement after solve by [False 5.1 #21 e]

h) If A is a 2×2 matrix then A must have 2 linearly independent (real) eigenvectors

Circle True OR provide a counterexample False like a shear matrix $A = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$, which has only the x-axis from clicker questions on eigenvector decomposition (5.6) part 2

i) If the largest eigenvalue equals 1, then the trajectory diagram would always have the populations dying off along that eigenvector.
Circle True OR provide a counterexample

False - we have stability here, so a counterexample would be a rough sketch like that from the glossary or eigenvector decomposition clickers (5.6) part 1: