

Fill-In Guide 2 for Linear Algebra—Bring to Class

Material in this class builds on itself so try to fill in the relevant items below as we cover them to help you internalize them and provide you with a working guide that you can use to help with activities such as handwrites, group reviews, and group debriefs during class as well as activities out of class such as practice quizzes, card sorts, problem sets and studying for exams. Some items are from earlier modules to help you make connections or compare and contrast with earlier material.

I want you to understand and internalize the material and I'm happy to help in and out of class!

| topic | first appeared | item numbers below |
|---|-----------------|-----------------------------|
| associativity | module 2 in 2.1 | 1 |
| column space | module 2 in 2.8 | 2 |
| determinants | module 4 in 3.1 | 37–40 |
| dot product (inner product) | module 3 in 6.1 | 5 |
| eigenvalues and eigenvectors | module 4 in 5.1 | 41–52 |
| equation of the line and slope a vector is on | module 1 in 1.3 | 53 |
| geometry of $t\vec{u}$ and $t\vec{u} + \vec{v}$ | module 1 in 1.3 | 54–55 |
| homogeneous | module 1 in 1.5 | 6–8 |
| invertible matrix theorem | module 2 in 2.3 | 9–10, 56 |
| length (norm of a vector) | module 3 in 6.1 | 11 |
| $\lim_{n \rightarrow \infty} a^n$ | prior classes | 57 |
| linear transformations | module 3 in 1.8 | 3–4, 12, 17–30, 32–35 |
| matrix multiplication | module 2 in 2.1 | 14–15 |
| matrix-vector multiplication | module 1 in 1.4 | 13 |
| nullspace | module 2 in 2.8 | 58–59 |
| orthogonality | module 3 in 6.1 | 16 |
| span | module 1 in 1.3 | 31 |
| summation notation | prior classes | 60 |
| transpose of a matrix | module 2 in 2.1 | 36 |

Module 3 and Related Items

- 1) associativity for multiplication—algebraic condition

- 2) column space of A—example of solving for the entire space as well as a basis

- 3) dilation by nonzero—input-output diagram example

- 4) dilation by nonzero—matrix representation example

- 5) dot product of 2 vectors—example

- 6) homogeneous coordinates—example of a matrix
- 7) homogeneous system—definition
- 8) homogeneous system—example of an augmented matrix
- 9) invertible matrix theorem—theorem statements in module 2
- 10) invertible matrix theorem—additional theorem statement in module 3
- 11) length or norm of a vector—how can we compute it?
- 12) linear transformation—definition
- 13) multiply a matrix by a column vector—example
- 14) multiply two 2×2 matrices—example

- 15) multiply two 3×3 matrices—example

- 16) orthogonality—how can we compute when 2 vectors are orthogonal?

- 17) projection onto a line—input-output diagram example

- 18) projection onto a line—matrix representation example

- 19) projection onto a plane—input-output diagram example

- 20) projection onto a plane—matrix representation example

- 21) range or output of a linear transformation—how to find it

- 22) reflection about a line—input-output diagram example

- 23) reflection about a line—matrix representation example

24) reflection about a plane—input-output diagram example

25) reflection about a plane—matrix representation example

26) rotation about $\vec{0}$ —input-output diagram example

27) rotation about $\vec{0}$ —matrix representation example

28) rotation about a point other than the origin—matrix representation example

29) rotation about a line—input-output diagram example

30) rotation about a line—matrix representation example

31) span of column vectors of a matrix—example

32) shear—input-output diagram example

33) shear—matrix representation example

34) translation—input-output diagram example

35) translation—matrix representation example

36) transpose of a matrix—example

Module 4 and Related Items (Some Related Items are Above)

37) determinant of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

38) determinant via cofactor or Laplace expansion—example

39) determinant of 2×2 matrix—what is a geometric interpretation?

40) determinant of 3×3 matrix—what is a geometric interpretation?

41) eigenvalue—characteristic equation definition

42) eigenvalue—definition

43) eigenvalue—example by-hand

44) eigenspace—example by-hand of solving for the entire space as well as a basis

45) eigenvector—definition

46) eigenvector—how to find an eigenvector

47) eigenvector—input-output diagram example

48) eigenvector decomposition example

49) eigenvalue/eigenvector long term behavior (behavior, eventual ratio of populations, rate of change...)—
how to tell from the eigenvector decomposition

50) eigenvalue/eigenvector trajectory diagram for long-term die off

51) eigenvalue/eigenvector trajectory diagram for long-term growth

52) eigenvalue/eigenvector trajectory diagram for long-term stability

53) equation of the line and slope a vector $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ in \mathbb{R}^2 is on

54) geometry of $t\vec{u}$ —example of a sketch that includes the original vector

55) geometry of $t\vec{u} + \vec{v}$ —example of a sketch that includes the original vectors

56) invertible matrix theorem—additional theorem statements in module 4

57) $\lim_{n \rightarrow \infty} a^n$ —examples where $a < 1$, $a = 1$ and $a > 1$

58) null space of A —definition

59) null space of A —example of solving for the entire space as well as a basis

60) summation notation—example