Linear Algebra Introduction: Evelyn Boyd Granville's Favorite Challenge

Evelyn Boyd Granville in 1997

Evelyn Boyd Granville was the second black woman we know of to receive her PhD in mathematics. Dr. Granville's original research related to complex numbers but she also worked on numerous space missions, including Project Mercury, the first manned space flight program: *I can say without a doubt that this was the most interesting job of my lifetime - to be a member of a group responsible for writing computer programs to track the paths of vehicles in space* (Granville, 1989). In this worksheet we will explore topics related to her favorite challenge.

My favorite challenge to teachers and children is to solve the following problem using three different methods: Rabbits and chickens have been placed in a cage. You count 48 feet and seventeen heads. How many rabbits and how many chickens are in the cage? (Granville, 2007)

Let x = the number of rabbits and y = the number of chickens

1. In terms of x and y, how many heads are there?
2. In terms of x and y, how many feet are there?
3. Solve these equations for x and y using at least three different methods.

Extensions

4. Given a certain number of heads and feet, must a mathematical solution for the numbers of rabbits and chickens always exist? Explain why or find a counterexample.
5. Say we have a system of real-life equations modeled via

$$x + ky = 0$$
$$kx + y = 0$$

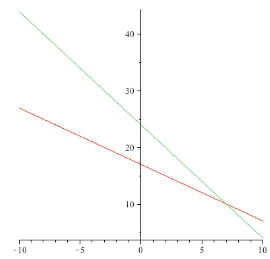
   a) Solve these equations for x and y.
   b) Must a mathematical solution for x and y always exist? Explain why or find a counterexample

c) Solutions for Evelyn Boyd Granville's Favorite Challenge Activity Sheet

Method 1
Chickens have 2 feet. Rabbits have 4 feet. Initially, we draw $17 \times 2 = 34$ feet and so $48 - 34 = 14$ feet remain. These remaining feet must belong to the rabbits. Once they are added to the drawing, 10 heads have two feet and are chickens, and the remaining 7 heads with four feet are rabbits.

Additional Methods Let x = the number of rabbits and y = the number of chickens. Then we have $x + y = 17$ heads and $4x + 2y = 48$ feet. Students can solve this problem a number of ways, such as

- Substitution: Since $y = 17 - x$ then $48 = 4x + 2y = 4x + 2(17 - x) = 2x + 34$, and so $14 = 2x$. Thus $x = 7$, and substituting into the first equation yields $y = 10$.
- Graphical intersection:



- Reducing the Augmented Matrix or Invertible Matrix Methods

$$\begin{matrix} 1 & 1 & 17 \\ 4 & 2 & 28 \end{matrix} \quad \text{reduces to} \quad \begin{matrix} 1 & 0 & 7 \\ 0 & 1 & 10 \end{matrix}$$

or

$$\begin{matrix} 1 & 1 \\ 4 & 2 \end{matrix} \quad \text{has inverse} \quad \begin{matrix} -1 & \frac{1}{2} \\ 2 & -\frac{1}{2} \end{matrix}$$

so the inverse can be applied to the column vector (17,28) to obtain the solution.

Extensions As above, let x = the number of rabbits and y = the number of chickens so that we have $4x + 2y$ feet and $x + y$ heads.

- Setting the number of heads and feet equal to each other reduces to the equation $3x = -y$, so there are mathematical solutions.
- If there are an equal number of rabbits and chickens, then $x = y$. Substituting for y in the equations yields $6x$ feet and $2x$ heads. As long as the ratio of feet to heads is 3 to 1, then there are mathematical solutions. In addition, the number of heads must be divisible by 2. Otherwise, the number of feet will be odd, and we will end up with fractions of chickens and rabbits, such as when we have 51 feet and 17 heads, which result in 17/2 rabbits and 17/2 chickens.
- Solutions must always exist mathematically. The lines have different slopes, so they will intersect. Alternatively, the coefficient matrix for the system has determinant -2 and so it is invertible.