Multiplication of a Matrix $A_{m\times n}$ and a Vector $\vec{x}_{n\times 1}$: $A\vec{x} = \vec{b}_{m\times 1}$

linear combinations of <u>columns</u> of A using the weights from $\vec{x}$:

$A\vec{x} = x_1 \text{col1}A + x_2 \text{col2}A + \dots$

$$\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}\begin{bmatrix}7\\8\\9\end{bmatrix} = 7\begin{bmatrix}1\\4\end{bmatrix} + 8\begin{bmatrix}2\\5\end{bmatrix} + 9\begin{bmatrix}3\\6\end{bmatrix} = \begin{bmatrix}7\cdot 1 + 8\cdot 2 + 9\cdot 3\\ 7\cdot 4 + 8\cdot 5 + 9\cdot 6\end{bmatrix}$$

dot products of <u>rows</u> of A using $\vec{x}$: $A\vec{x} = \begin{bmatrix}\text{row1}A\cdot\vec{x}\\ \text{row2}A\cdot\vec{x}\\ \vdots\end{bmatrix}$

$$\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}\begin{bmatrix}7\\8\\9\end{bmatrix} = \begin{bmatrix}\begin{bmatrix}1 & 2 & 3\end{bmatrix}\cdot\begin{bmatrix}7\\8\\9\end{bmatrix}\\ \begin{bmatrix}4 & 5 & 6\end{bmatrix}\cdot\begin{bmatrix}7\\8\\9\end{bmatrix}\end{bmatrix}$$ = sum product of entries

Multiple representations allow us to go back and forth between a matrix equation, vector equation, system of equations, and augmented matrix