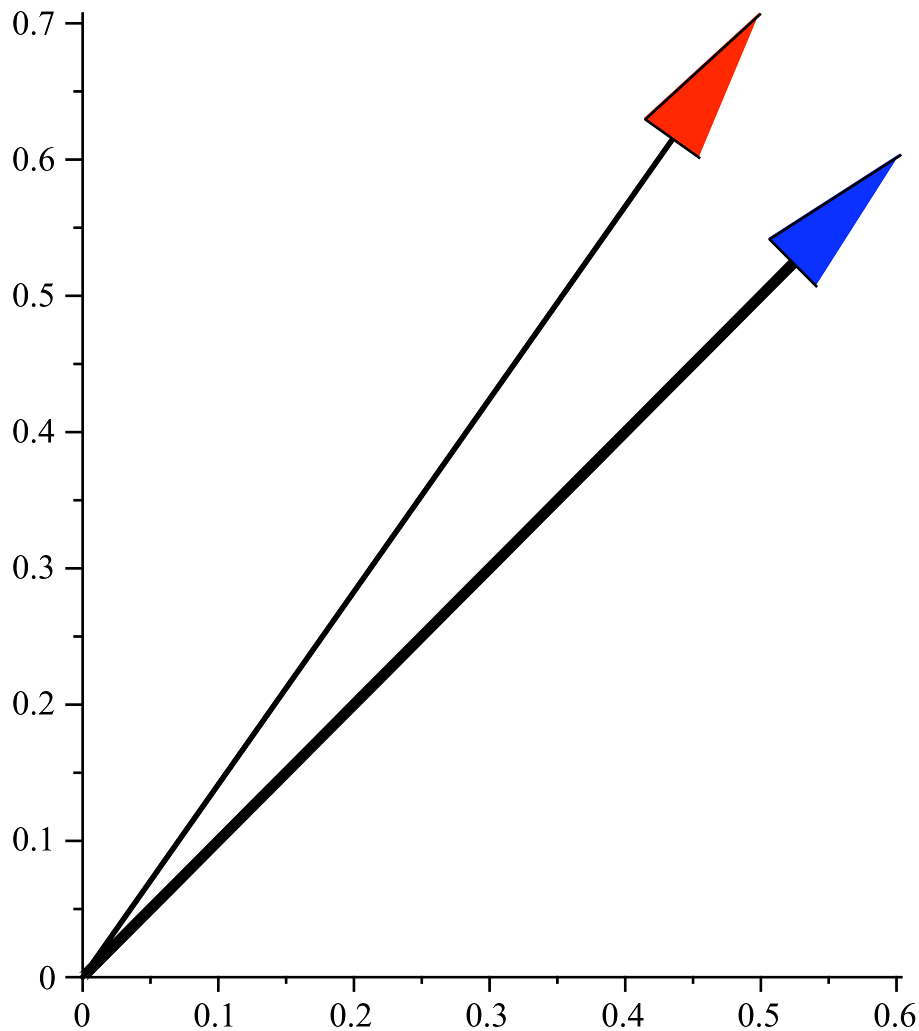
$with(LinearAlgebra) : A := Matrix([[(\cos(\text{theta}))\verb|^|2/((\cos(\text{theta}))\verb|^|2 + (\sin(\text{theta}))\verb|^|2), (\cos(\text{theta}) * \sin(\text{theta}))/((\cos(\text{theta}))\verb|^|2 + (\sin(\text{theta}))\verb|^|2)], [(\cos(\text{theta}) * \sin(\text{theta}))/((\cos(\text{theta}))\verb|^|2 + (\sin(\text{theta}))\verb|^|2), ((\sin(\text{theta}))\verb|^|2)/((\cos(\text{theta}))\verb|^|2 + (\sin(\text{theta}))\verb|^|2)]]);$

$$\begin{bmatrix} \frac{\cos(\theta)^2}{\cos(\theta)^2 + \sin(\theta)^2} & \frac{\cos(\theta)\sin(\theta)}{\cos(\theta)^2 + \sin(\theta)^2} \\ \frac{\cos(\theta)\sin(\theta)}{\cos(\theta)^2 + \sin(\theta)^2} & \frac{\sin(\theta)^2}{\cos(\theta)^2 + \sin(\theta)^2} \end{bmatrix} \tag{1}$$

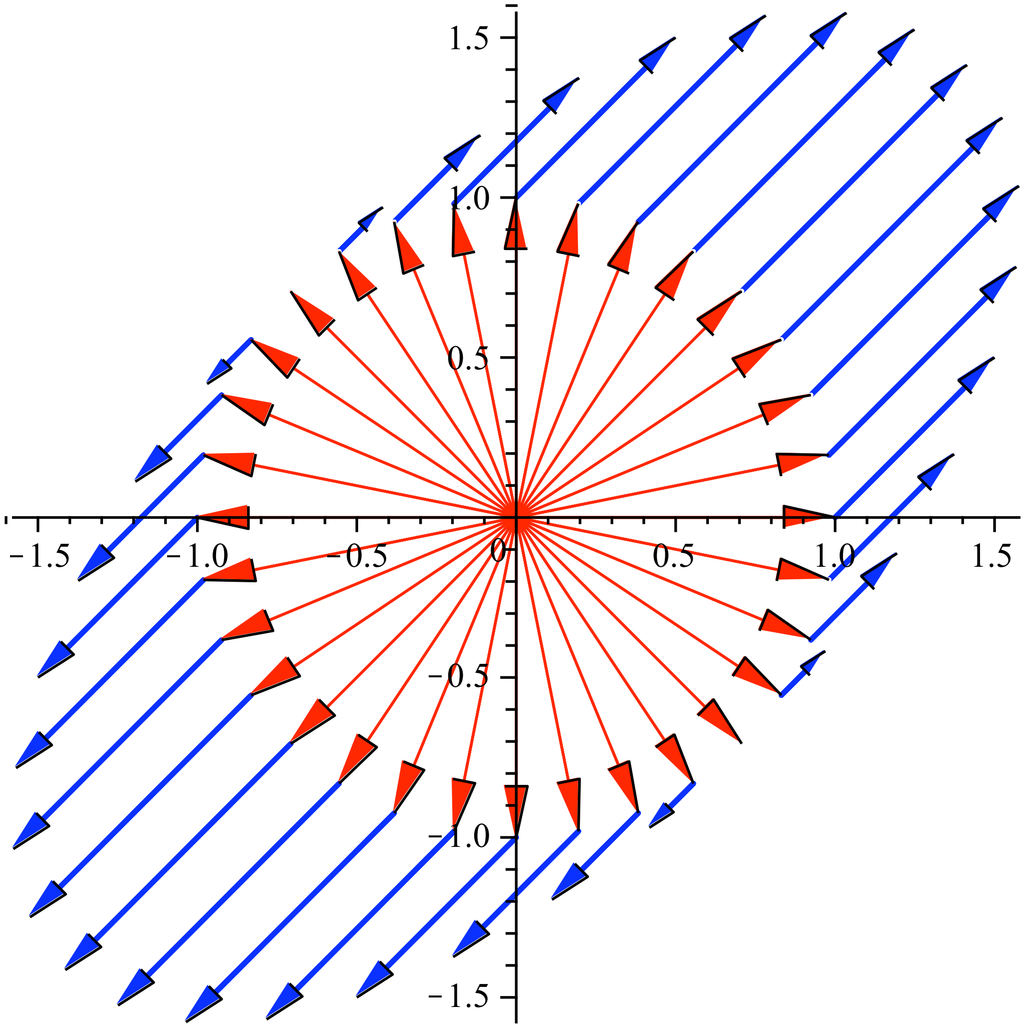
A represents a matrix that projects vectors onto the line through the origin that makes an angle of theta degrees with the positive x-axis. Ie this projects the shadow that an object makes as light rays come in perpendicular to the line given by y = tan(theta) x.

Here is what happens when theta is $\frac{\text{Pi}}{4}$. First, here is the input (red) and output (blue) picture for one vector. Notice that the red vector is projected onto the y=tan(theta)x line via its shadow.

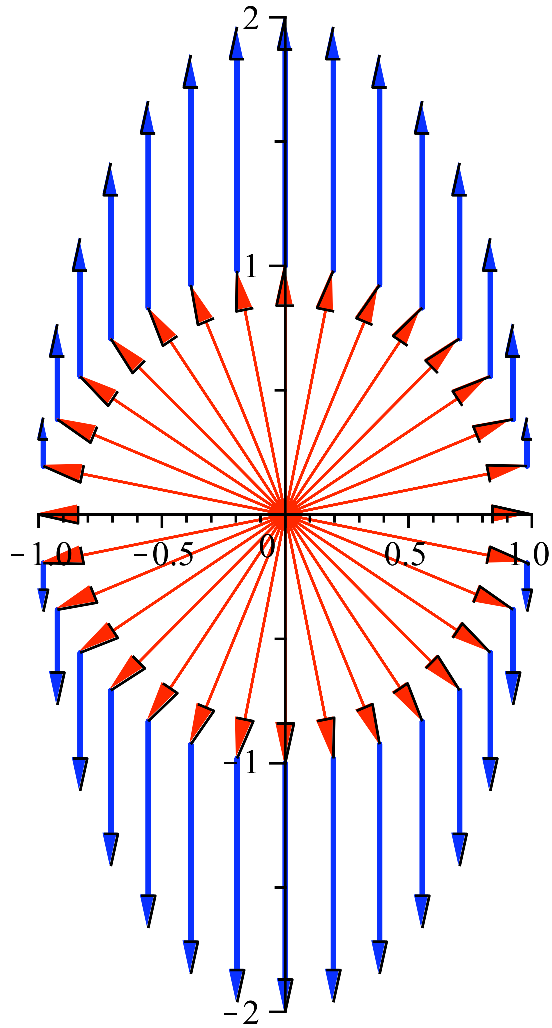
$Drawvec(w, [A.w, headcolor = blue, thickness = 4]);$



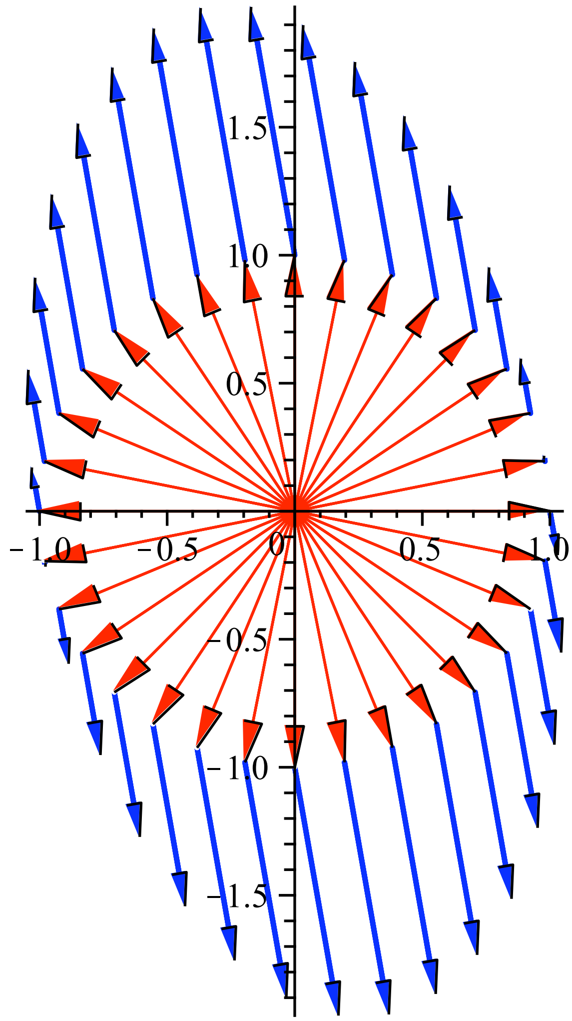
We place the output vectors in blue at the tail of the input vectors for ease of visualizing them all at once.



When theta is $\frac{\text{Pi}}{2}$



When theta is $\frac{\text{Pi}}{1.8}$



We can identify the eigenvectors by looking for vectors which stay on the same line through the origin.

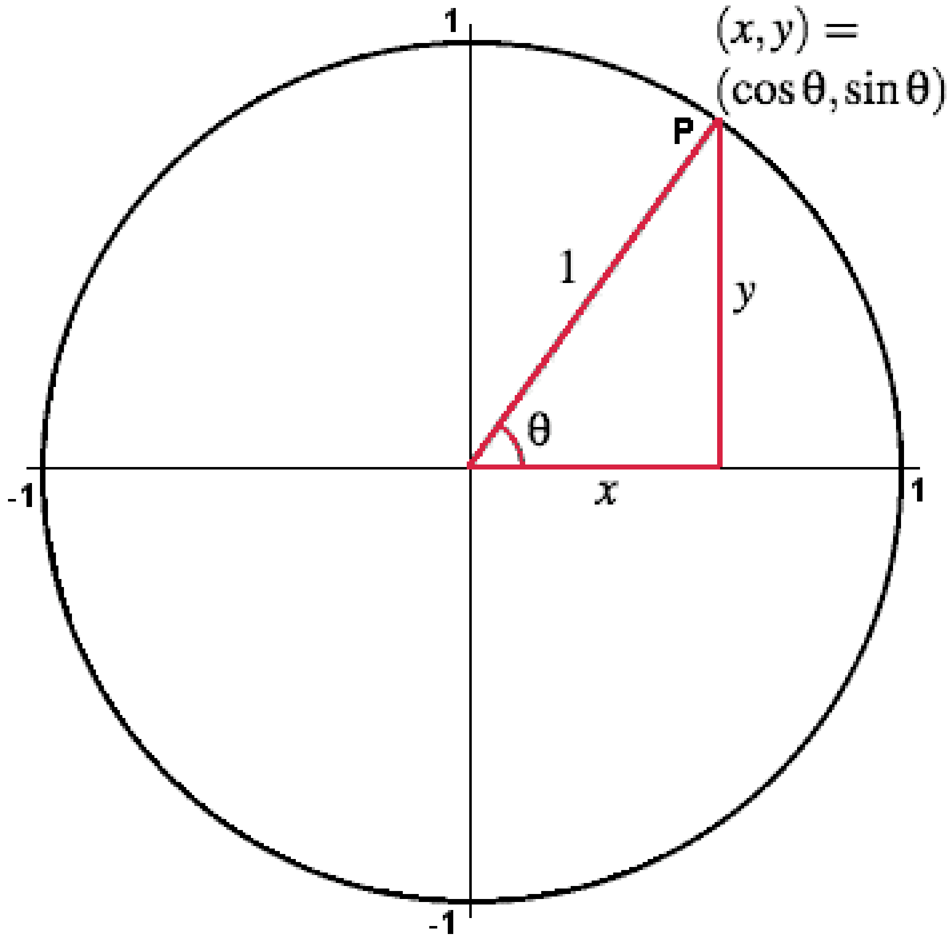
Eigenvectors(*A*);

```
Error, (in LinearAlgebra:-LA_Main:-Eigenvectors) expecting either Matrices of rationals, rational functions, radical functions, algebraic numbers, or algebraic functions, or Matrices
```

```
of complex(numeric) values
```
Eigenvalues(*A*);

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (2)$$

The eigenvector of 1 corresponds to vectors on the line of projection



ie

$$\begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} \quad (3)$$

that has full shadow and the eigenvector of 0 corresponds to vectors perpendicular so we take the negative reciprocal $\begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}$ that has no shadow. Each of these is a basis representative for their respective lines. All other vectors are moved off their original lines onto the y=tan(theta) x line. So now we form P as the matrix of eigenvectors:

$P := Matrix([[\cos(\text{theta}), -\sin(\text{theta})],$
$[\sin(\text{theta}), \cos(\text{theta})]]);$

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \tag{4}$$

P is a rotation matrix with a counterclockwise rotation by theta. The inverse matrix is a clockwise rotation by theta.

$Diag := Matrix([[1, 0], [0, 0]]);$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{5}$$

The Diag matrix is the matrix that projects onto the x-axis, ie when theta is is 0.

P.Diag.MatrixInverse(*P*);

$$\begin{bmatrix} \frac{\cos(\theta)^2}{\cos(\theta)^2+\sin(\theta)^2} & \frac{\cos(\theta)\sin(\theta)}{\cos(\theta)^2+\sin(\theta)^2} \\ \frac{\cos(\theta)\sin(\theta)}{\cos(\theta)^2+\sin(\theta)^2} & \frac{\sin(\theta)^2}{\cos(\theta)^2+\sin(\theta)^2} \end{bmatrix} \tag{6}$$

A is the projection onto the y=tan(theta) line. Using diagonalization, this is also the same as first rotating a point clockwise by theta, using the projection onto the x-axis, and then rotating counterclockwise back where we began. This type of visualization will be very useful for computer graphics.