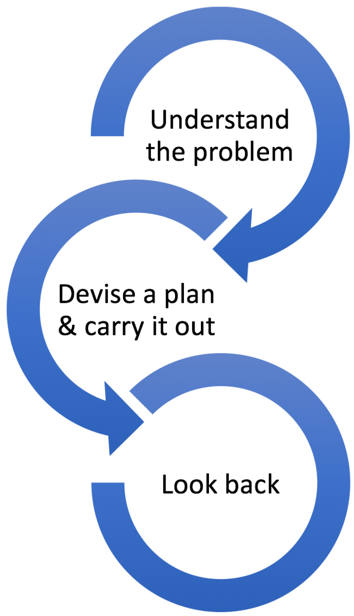
Problem Set 1

Instructions: You may work alone or in a group of up to 2 people and turn in one per group but each group must complete and write up their work in their own words. Your group collates your problem set into one multipage PDF for submission in one of your ASULearn assignments. Feel free to talk to me (I am always happy to help!) or each other if you are stuck on this assignment, but be sure to acknowledge any sources or people, aside from your partner or me. If you know how to do a problem and are asked for help, try to give hints rather than the solution. Use only what we have covered so far and in the language of our class. Explain in your own words, execute your own adaptions to the Maple file **prs1s22.mw** (and include any code and output in your PDF), create your own pictures (if any), and give any references you used (if any), including each other.



Mathematics, you see, is not a spectator sport. [George Polya, *How to Solve it*]

1. List your preferred name(s). If you are turning in with a partner, list both names (and turn in to one of your ASULearn assignments)

2. Consider the algebra and geometry of the following system:

$$\begin{aligned} x - 2y - z &= 4 \\ -2x + 4y - 5z &= 6 \\ 2x - 4y + 12z &= -20 \end{aligned}$$

   (a) Solve the system by-hand using strict Gaussian elimination—no scaling of rows—and then back substitution. Show work and reduction steps to row echelon form and then show work for back substitution to write out any solutions. If there are infinite solutions, parameterize them.

   (b) Download the **prs1s22.mw** file and modify it to use ReducedRowEchelonForm in Maple. Include your code and output.

   (c) Modify the Maple file to use implicitplot3d on the original rows. Include the code and output.

   (d) Do the above 3 methods yield the same solutions? Compare and contrast what each shows and how they do or do not relate to each other.

(e) Modify the spacecurve commands in **prs1s22.mw** in order to plot the column vectors from the original coefficient matrix rather than my favorite example that is shown there. Provide your code.

(f) Can we turn the plot "head on" so that the 3 column vectors in the coefficient matrix for the original system are in the same plane? Explain and include the Maple plot or sketch it.

(g) What is the span of the 3 vectors geometrically? Explain how it relates to the display of the Maple spacecurve graphs.

(h) Are the 3 column vectors in the coefficient matrix linearly independent? Explain why or why not.

3. Concrete mix, which is used in jobs as varied as making sidewalks and building bridges, is composed of five main materials: cement, water, sand, gravel, and fly ash. By varying the percentages of these materials, mixes of concrete can be produced with differing characteristics. For example, the water-to-cement ratio affects the strength of the final mix, the sand-to-gravel ratio affects the "workability" of the mix, and the fly-ash-to-cement ratio affects the durability. Since different jobs require concrete with different characteristics, it is important to be able to produce custom mixes.

   Assume you are the manager of a building supply company and plan to keep on hand three basic mixes of concrete from which you will formulate custom mixes for your customers. The basic mixes have the following characteristics:

	Super-Strong Type S	All-Purpose Type A	Long-Life Type L
Cement	20	18	12
Water	10	10	10
Sand	20	25	15
Gravel	10	5	15
Fly ash	0	2	8

   Each measuring scoop of any mix weighs 60g, and the numbers in the table above give the breakdown by grams of the components of the mix. We can represent any mixture by a vector $\begin{bmatrix} c \\ w \\ s \\ g \\ f \end{bmatrix}$ in $\mathbb{R}^5$ representing the amounts of cement, water, sand, gravel, and fly ash in the final mix.

   (a) What is our book's definition (or the glossary) of a linear combination of vectors?

   (b) What is our book's definition (or the glossary) of the span of a set of vectors?

   (c) Is the custom mix $S + A + L$ in the span of $\{S, A, L\}$? Explain.

   (d) Compare the strength of the custom mix $S + A + L$ to the strength of the L mix. Which is stronger (low water to cement ratio)? Show reasoning/work. This is one example of a practical interpretation of vector coordinates.

   (e) A customer requests a custom mix with the following proportions of of cement, water, sand, gravel, and fly ash: 12: 12: 12: 12: 12. Is it possible to make this mix using only S, A and L? Show Maple code and output adapted from **prs1s22.mw** to find the weights (in scoops/parts of scoops) of the basic mixes (S, A, and L) needed to create this mix, if it is possible, and explain why or why not from this output.

(f) What is our book's definition (or the glossary) of linearly independent vectors?

(g) Let $U = \begin{bmatrix} 12 \\ 12 \\ 12 \\ 12 \\ 12 \end{bmatrix}$ and $V = \begin{bmatrix} 1 \\ 23 \\ 12 \\ 12 \\ 12 \end{bmatrix}$. Is $\{S, A, L, U, V\}$ linearly independent? Why or why not? Show Maple code and output or by-hand work plus your reasoning.

(h) What practical advantage would $\{S, A, L, U, V\}$ being linearly independent have? For example, think about the mixes you can make using 5 linearly independent vectors compared to if you had 5 vectors that were not l.i. (i.e. dependent vectors). Say something beneficial about l.i. in this context.

(i) Consider negative weights to explain why are there are still mixes in $\mathbb{R}^5$ that cannot physically be produced from $\{S, A, L, U, V\}$ even though they can be produced mathematically?

4. Skim 1.6 or other parts of the book or search the internet for a real-life application we haven't yet covered related to one or more of the following topics: system of linear equations, vectors, matrix equations, linear combinations, or span. Name the application, the topics from this list that connect, and the source.

5. Proper citations or smiley face or similar, look back and smiley face or similar, collate to one PDF:

   (a) Regardless of whether you talked to others or used other sources, be sure that your project consists of products that you and your partner (if any) create yourselves and in your own words. Give proper credit to anyone you talked to, other than if you are turning this in with a partner or talked to me, and give proper credit to any source citations. If this doesn't apply or you already completed this above, then write a smiley face, N/A, or similar.

   (b) Look back and then write a smiley face or similar: take the time to reflect and ensure that you have answered all parts of the questions, showed work, included your by-hand and Maple work, and explained your reasoning in your own group's words (there is no need for full sentences). Be sure you are using only what we have covered so far and in the language of our class. A good rule of thumb in deciding how much to write is to write enough so that a classmate who hasn't yet solved the problem could understand what you are doing and why—how you reached your conclusion from the computation or example—so they are persuaded of its validity by the logic and clarity of your reasoning. Write a smiley face or similar.

   (c) Collate your work into one PDF for submission to the ASULearn assignment. If you are turning this in with a partner, turn in one complete project writeup in **one of your ASULearn assignments**. Electronically, you can append PDFs you create from Maple to the end of your other PDFs, like by using Preview on a Mac or PDFsam on a PC. Or, if you are physically printing, you can print your Maple work and then append it to the end of handwritten work and then scan it all in to one file. If you have a phone or tablet, apps like Adobe Scan or CamScanner can work well to scan work to one full size multipage PDF. You can also use many printers or photo copiers to scan to PDFs—the school library lists that as an option and they can help:
   `https://library.appstate.edu/services-search/print-zone-tech-help`.