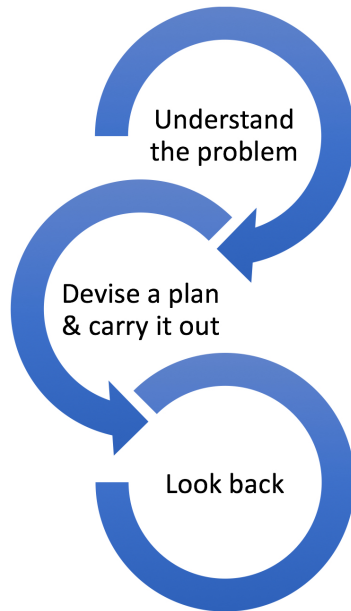


## Problem Set 3

**Instructions:** You may work alone or in a group of up to 2 people and turn in one per group but each group must complete and write up their work in their own words. Your group collates your problem set into one single PDF for submission in the ASULearn assignment. Feel free to talk to me (I am always happy to help!) or each other if you are stuck on this assignment, but be sure to acknowledge any sources or people, aside from your partner or me. Use only what we have covered so far and in the language of our class. If you know how to do a problem and are asked for help, try to give hints rather than the solution. Explain in your own words, execute your own adaptations to the Maple file **prs3s22.mw** (and include any code and output), create your own pictures (if any), and give any references you used (if any), including each other.



*Mathematics, you see, is not a spectator sport.* [George Polya, *How to Solve it*]

1. List your preferred name(s). If you are turning this in with a partner, list both names.

### 2. Linear Transformations of a Car and Hotel

#### Transforming a Car in Homogeneous Coordinates

(a) Execute the commands in the Maple file **prs3s22.mw** (don't forget to execute the packages!) for Translating a Car in Homogeneous Coordinates. We are plotting the tips of the vectors in

the homogeneous version of the car and translating the car using the translation  $\begin{bmatrix} 1 & 0 & 16 \\ 0 & 1 & -24 \\ 0 & 0 & 1 \end{bmatrix}$ .

Here the original car is red with thicker connections and the translated car is blue. Rotate the plot. Do both cars lie in the  $z = 1$  plane?

(b) Next, modify the part(b) translation matrix in Maple

**ModifyTranslation:=Matrix([[1,0,16],[0,1,-24],[0,0,1]]);**

to create your own—different—translation and plot. Include the modified code for the new translation and the new plot of the cars in your PDF.

(c) Stay in homogeneous coordinates as you modify the identity linear transformation (part(c) in Maple) to something that is different than the above and does something interesting to the car. Be sure to stay in homogeneous coordinates. Include the modified code for the matrix representation of the linear transformation and the new plot of the cars in your PDF.

### Transforming a Hotel

- (d) Execute the commands for Transforming a Hotel for Guess1 in part (d) in Maple. What kind of linear transformation is this and what does it do to the hotel (input is thicker red and output is blue)? Fill in both blanks for this linear transformation:  
\_\_\_\_\_ (select one or more that best applies: about, across, onto, and/or by) \_\_\_\_\_?
- (e) Continue with Guess 2. What kind of linear transformation is this and what does it do to the hotel (input is thicker red and output is blue)? Fill in both blanks for this linear transformation:  
\_\_\_\_\_ (select one or more that best applies: about, across, onto, and/or by) \_\_\_\_\_?

### 3. Hill Cipher

Assume that we intercept a number of items, as follows:

- The following text sent: *A system of inconsistent equations walks into a doctors office*
- A string of coded numbers as a reply:  
-2, -5, -12, -13, 9, 2, -3, -3, 25, 10, 6, 0, 15, 6, 13, 4, 8, -6, 38, 19
- The beginning of the decoded message: *Chan*
- The fact that a 2x2 decoding matrix was used in the Hill Cipher

What does the doctor prescribe? We'll investigate whether the rest of the message can be decoded:

- (a) Set up two matrix vector equations for the decoding matrix  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$  using the intercepted part of the message, either by-hand or in Maple and annotate as below:

Define the decoding matrix using generic variables  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\begin{bmatrix} -2 \\ -5 \end{bmatrix} \text{ decodes to the vector corresponding to "ch" via } \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} -2 \\ -5 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$$

Now we set up the next part of the string  $\begin{bmatrix} -12 \\ -13 \end{bmatrix}$  decoding to the vector corresponding to "an"—you create the vector from the standard digitized alphabet (space is 0, a is 1...) as well as the matrix vector equation and annotate the linear algebra like I did just above, but for this part of the message.

- (b) Multiply the systems by-hand or in Maple using **prs3s22.mw**. Show your by-hand work or Maple code and output.
- (c) From setting equal each corresponding entry in part (b), write down the 4 equations in the 4 unknowns  $a, b, c, d$ .
- (d) Solve this system. It's linear—so we can solve it using a variety of methods, like a 4x5 augmented matrix with columns  $abcd$  and an equals column with first row  $[-2,-5,0,0,3]$ . Show your by-hand work or Maple code and output by modifying **prs3s22.mw**.
- (e) How many solutions do we have? 0, 1 or infinite solutions for  $a, b, c$ , and  $d$ ?
- (f) If we have solutions, put them into  $\text{DecodingMatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  (careful about which numbers are in which spots) and take the determinant.

A determinant of 1 or -1 can be quite useful in real-life, not only because we need an invertible matrix, but also so we don't end up with fractions in our decoding or encoding matrix from

$$\begin{bmatrix} \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\ -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}.$$

Is the determinant  $\pm 1$ ? Respond and show the matrix and the determinant.

- (g) If we have solutions, decode (by-hand or in Maple by modifying **prs3s22.mw**. Notice that the coded string goes in to the CodedMessage as column vectors). Show your by-hand work or Maple code and output.
- (h) Then translate back to letters by reading down the columns to explain what the doctor prescribed. If there are no solutions to the cipher, then explain why the system is inconsistent.

#### 4. Orthogonality in my Favorite Example

Consider the vectors  $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$  and  $\vec{v} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$  in  $\mathbb{R}^3$ .

- (a) Show that  $\vec{w} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$  is orthogonal to  $\vec{u}$ . You can either do this by-hand, showing work, or in Maple in **prs3s22.mw**, showing your code and output.

- (b) Show that  $\vec{w}$  is also orthogonal to  $\vec{v}$  (your choice of by-hand or Maple but show work/Maple).

- (c) Recall that the span of a set of vectors is all the linear combinations of them, over all weights,

i.e. all the vectors we can create via  $c_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ . As in chapter 1, we can set up a generic

vector  $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$  to see when it is in the span. Use strict Gaussian (replacements like  $r'_2 = -2r_1 + r_2$

but no scaling) to reduce the matrix  $\begin{bmatrix} 1 & 4 & x \\ 2 & 5 & y \\ 3 & 6 & z \end{bmatrix}$  to row echelon form only via strict Gaussian

by-hand or in Maple in **prs3s22.mw**. Show your by-hand work or your Maple code and output.

- (d) Use part c) to write an equation  $\_\_x + \_\_y + \_\_z = 0$  for the plane spanned by  $\vec{u}$  and  $\vec{v}$ , i.e. fill in the blanks. This is a plane since we have 2 vectors that are not on the same line as they are not multiples.

- (e) Compare each coordinate of  $\vec{w}$  (the  $x$ -coordinate, the  $y$ -coordinate and the  $z$ -coordinate) to the corresponding coefficients of the equation of the plane in part d). What do you notice?

- (f) In Maple, modify the coefficients of the plane in the implicitplot3d command in **prs3s22.mw** to plot the plane along with the vector  $\vec{w}$ . Turn the plot so that the orthogonality is shown. Show your code and output.

#### 5. Proper citations or smiley face or similar, look back and smiley face or similar, collate to one PDF:

- (a) Regardless of whether you talked to others or used other sources, be sure that your project consists of products that you and your partner (if any) create yourselves and in your own words. Give proper credit to anyone you talked to, other than if you are turning this in with a partner or talked to me, and give proper credit to any source citations. If this doesn't apply or you already completed this above, then write a smiley face, N/A, or similar.

- (b) Look back and then write a smiley face or similar: take the time to reflect and ensure that you have answered all parts of the questions, showed work, included your by-hand and Maple work, and explained your reasoning in your own group's words (there is no need for full sentences).

Be sure you are using only what we have covered so far and in the language of our class. A good rule of thumb in deciding how much to write is to write enough so that a classmate who hasn't yet solved the problem could understand what you are doing and why—how you reached your conclusion from the computation or example—so they are persuaded of its validity by the logic and clarity of your reasoning. Write a smiley face or similar.

- (c) Collate your work into one PDF for submission to the ASULearn assignment. If you are turning this in with a partner, turn in one complete project writeup in **one of your ASULearn assignments**. Electronically, you can append PDFs you create from Maple to the end of your other PDFs, like by using Preview on a Mac or PDFsam on a PC. Or, if you are physically printing, you can print your Maple work and then append it to the end of handwritten work and then scan it all in to one file. If you have a phone or tablet, apps like Adobe Scan or CamScanner can work well to scan work to one full size multipage PDF. You can also use many printers or photo copiers to scan to PDFs—the school library lists that as an option and they can help:

<https://library.appstate.edu/services-search/print-zone-tech-help>.