# Problem Set 4

Instructions: You may work alone or in a group of up to 2 people and turn in one per group but each group must complete and write up their work in their own words. Your group collates your problem set into one single PDF for submission in the ASULearn assignment. Feel free to talk to me (I am always happy to help!) or each other if you are stuck on this assignment, but be sure to acknowledge any sources or people, aside from your partner or me. Use only what we have covered so far and in the language of our class. If you know how to do a problem and are asked for help, try to give hints rather than the solution. Explain in your own words, execute your own adaptions to the Maple file **prs4s22.mw** (and include any code and output), create your own pictures (if any), and give any references you used (if any), including each other.
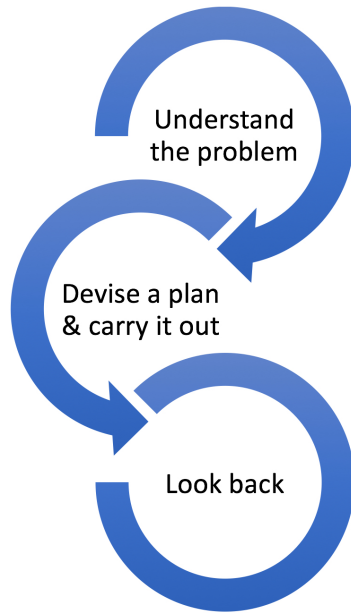
Mathematics, you see, is not a spectator sport. [George Polya, *How to Solve it*]

1. List your preferred name(s). If you are turning this in with a partner, list both names.

2. Determinants

   (a) Using random matrices in **prs4s22.mw**, compute
   Determinant(A1.B1)-Determinant(A1)*Determinant(B1), where A1 and B1 are two Maple-randomized 5×5 matrices
   Provide your code and output of the random matrices and the determinant.

   (b) What does this imply about the determinant of the product of matrices?

   (c) Look up a way that determinants are applied to something you are interested in. For example, you might search for
   determinants and eight queens (in game theory)
   determinants and volumetric strain (in geology)
   determinants of metric tensors (in mathematical physics/differential geometry)
   determinants quantum mechanics (in physical chemistry)
   matrix determinants in actuarial science
   or other applications. Provide a related reference source.

3. Eigenvalues, Eigenvectors and Dynamical Systems

   (a) What is our book's definition (or the glossary) of an eigenvalue?

   (b) What is our book's definition (or the glossary) of an eigenvector?

   (c) In Maple (**prs4s22.mw**) compute the eigenvalues and eigenvectors of $\begin{bmatrix} \frac{5}{10} & \frac{4}{10} \\ -\frac{125}{1000} & \frac{11}{10} \end{bmatrix}$. Provide your code and output.

   (d) By-hand, compute the eigenvalues of the same matrix and show work, including the characteristic equation and the quadratic formula to solve for the roots.

   (e) By-hand, compute the eigenvectors for one of the eigenvalues and show work for the null space of $A - \lambda I$. Keep any variables without pivots free, like we usually do.

   (f) Sketch an input vector and an output vector on one graph for an eigenvector connected to your by-hand work. Identify which is the input and which is the output.

   (g) Compare and contrast your by-hand work with Maple—what each shows and how they do or do not relate to each other. Explain.

   (h) Explain why the eigenvectors span all of $\mathbb{R}^2$ using either an argument using determinant and the what makes a matrix invertible theorem or the definition of span by augmenting with a generic vector and reducing. Show work and provide reasoning.

   (i) What is our book's definition (or the glossary) of an eigenvector decomposition?

   (j) Write out the eigenvector decomposition for the system.

   (k) Do the populations grow, die off, stabilize, or exhibit some other behavior in the long run for most starting positions? Annotate how you can tell from the eigenvector decomposition.

   (l) What is the eventual ratio of the populations the system tends to in the long run for most starting positions? Annotate how you can tell from the eigenvector decomposition.

   (m) For most starting positions, what is the yearly rate (growth rate, die off rate, or stability rate) in the long run? Annotate how you can tell from the eigenvector decomposition.

   (n) Sketch a by-hand plot of Maple's two eigenvectors.

   (o) Add to your plot to make it a trajectory plot by selecting one starting position in the 1st quadrant that is not on either eigenvector and demonstrating the long-term behavior of it.

   (p) What do you think might happen to the system in the longterm if some aspect of the model (such as birth rates or the predation rate) were to change slightly? Explain.

   (q) Look up a way that eigenvalues and eigenvectors are applied to something you are interested in. For example, you might search for
   eigenvalues and eigenvectors and sports analytics
   eigenvalues and eigenvectors stock and credit market
   eigenvalues and eigenvectors data science
   eigenvalues and eigenvectors machine learning
   eigenvalues and eigenvectors singular value decomposition
   eigenvalues and eigenvectors structural geology
   eigenvalues and eigenfunctions quantum mechanics
   eigenvalues and eigenfunctions quantum chemistry
   or other applications. Provide a related reference source.

4. Proper citations or smiley face or similar, look back and smiley face or similar, collate to one PDF:

   (a) Regardless of whether you talked to others or used other sources, be sure that your project consists of products that you and your partner (if any) create yourselves and in your own words. Give proper credit to anyone you talked to, other than if you are turning this in with a partner or talked to me, and give proper credit to any source citations. If this doesn't apply or you already completed this above, then write a smiley face, N/A, or similar.

   (b) Look back and then write a smiley face or similar: take the time to reflect and ensure that you have answered all parts of the questions, showed work, included your by-hand and Maple work, and explained your reasoning in your own group's words (there is no need for full sentences). Be sure that you are using only what we have covered so far and in the language of our class. A good rule of thumb in deciding how much to write is to write enough so that a classmate who hasn't yet solved the problem could understand what you are doing and why—how you reached your conclusion from the computation or example—so they are persuaded of its validity by the logic and clarity of your reasoning. Write a smiley face or similar.

   (c) Collate your work into one PDF for submission to the ASULearn assignment. If you are turning this in with a partner, turn in one complete project writeup in **one of your ASULearn assignments**. Electronically, you can append PDFs you create from Maple to the end of your other PDFs, like by using Preview on a Mac or PDFsam on a PC. Or, if you are physically printing, you can print your Maple work and then append it to the end of handwritten work and then scan it all in to one file. If you have a phone or tablet, apps like Adobe Scan or CamScanner can work well to scan work to one full size multipage PDF. You can also use many printers or photo copiers to scan to PDFs—the school library lists that as an option and they can help:
   https://library.appstate.edu/services-search/print-zone-tech-help.