

Rotation Matrix - Geometry

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$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\theta)x - \sin(\theta)y \\ \sin(\theta)x + \cos(\theta)y \end{bmatrix}$$

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$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}$$

Using the unit circle, we see that the output is on the $y = \tan(\theta)x$ line, ie the same length vector but θ degrees counterclockwise.

$$\begin{bmatrix} 2 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2\cos(\theta) \\ 2\sin(\theta) \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow$$

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Using the unit circle, we see that the output is on the $y = \tan(\theta)x$ line, ie the same length vector but θ degrees counterclockwise.

$$\begin{bmatrix} 2 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2\cos(\theta) \\ 2\sin(\theta) \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}$$

Rotation Matrix - Algebra

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\theta)x - \sin(\theta)y \\ \sin(\theta)x + \cos(\theta)y \end{bmatrix}$$

Rotation Matrix - Algebra

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\theta)x - \sin(\theta)y \\ \sin(\theta)x + \cos(\theta)y \end{bmatrix}$$

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} r\cos(\phi) \\ r\sin(\phi) \end{bmatrix} = \begin{bmatrix} r\cos(\theta)\cos(\phi) - r\sin(\theta)\sin(\phi) \\ r\sin(\theta)\cos(\phi) + r\cos(\theta)\sin(\phi) \end{bmatrix}$$

Rotation Matrix - Algebra

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\theta)x - \sin(\theta)y \\ \sin(\theta)x + \cos(\theta)y \end{bmatrix}$$

$$\begin{aligned} \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} r\cos(\phi) \\ r\sin(\phi) \end{bmatrix} &= \begin{bmatrix} r\cos(\theta)\cos(\phi) - r\sin(\theta)\sin(\phi) \\ r\sin(\theta)\cos(\phi) + r\cos(\theta)\sin(\phi) \end{bmatrix} \\ &= \begin{bmatrix} r\cos(\theta + \phi) \\ r\sin(\theta + \phi) \end{bmatrix} \end{aligned}$$

Transformations of \mathbb{R}^2

Rotation: $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ Dilation: $\begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}$ Horizontal Shear: $\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$

Projections: $y=x$ line: $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ x -axis: $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ y -axis: $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Reflections: $y=x$ line: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ x -axis: $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ y -axis: $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

- What important transformation is missing?

Transformations of \mathbb{R}^2

$$\text{Rotation Matrix: } \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$\text{Dilation Matrix: } \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}$$

$$\text{Horizontal Shear: } \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$$

$$\text{Projection Matrix: } \begin{bmatrix} \cos(\theta)^2 & \cos(\theta)\sin(\theta) \\ \cos(\theta)\sin(\theta) & \sin(\theta)^2 \end{bmatrix}$$

$$\text{Reflection Matrix: } \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{bmatrix}$$

- What important transformation is missing?

Translation Matrix: $\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + h \\ y + k \end{bmatrix}$

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$$\text{Translation Matrix: } \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + h \\ y + k \end{bmatrix}$$

$$\text{Translation Matrix: } \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + h \\ y + k \\ 1 \end{bmatrix}$$

$$\text{Rotation Matrix: } \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotate a Figure about the point $\begin{bmatrix} 4 \\ 9 \end{bmatrix}$:

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -9 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 & \dots & x_p \\ y_1 & \dots & y_p \\ 1 & \dots & 1 \end{bmatrix}$$