Rotation: $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ Dilation: $\begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}$ Horizontal Shear: $\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$

Projections: y=x line: $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ x-axis: $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ y-axis: $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Reflections: y=x line: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ x-axis: $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ y-axis: $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

Translation: $\begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x+h \\ y+k \\ 1 \end{bmatrix}$ Others: $\begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Rotate a Figure about the point $\begin{bmatrix} 4 \\ 9 \end{bmatrix}$:

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -9 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 & ... & x_p \\ y_1 & ... & y_p \\ 1 & ... & 1 \end{bmatrix}$$

Applications of Matrix Algebra:

Keeping a car on a racetrack

- In the rotation matrix we used orthogonal vectors, and unitized each of them by dividing by the length/norm, to avoid the size of the car changing with the speed
- Multiplication is not associative–fly off the road (Rotate Translate car)or stay on it (Translate Rotate car)

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Transforming Yoda

- Yoda was given in rows but our transformations and left multiplication use columns
- $(Rotation Yoda^T)^T$

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- $\underbrace{(Rotation\,Yoda^T)^T}_{\text{slow}} = (Yoda^T)^T Rotation^T = \underbrace{Yoda\,Rotation^T}_{\text{faster}}$
- So we make use of $(AB)^T = B^T A^T$ to save time—in this case we convert to right multiplication by $Rotation^T$

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  slow faster
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$$A_{2\times 2}(B_{2\times 2}D_{2\times 100}) = (A_{2\times 2}B_{2\times 2})D_{2\times 100}$$

slow faster

800 408