Given a matrix A, an eigenvector stays on the same line through the original which A was on, and so

$$A\begin{pmatrix}x\\y\end{pmatrix}=t\begin{pmatrix}x\\y\end{pmatrix}$$

for some real number t. Hence eigenvectors turn matrix multiplication into scalar multiplication.

Let's revisit a problem from earlier in the semester: Each year 6% of the urban population moves to rural areas and the remaining 94% stay in urban areas. Each year 9% of the rural population moves to urban areas and the remaining 91% stay in rural areas.

```
> A:=Matrix([[94/100,9/100],[6/100,91/100]]);
```

$$A := \begin{bmatrix}\frac{47}{50} & \frac{9}{100}\\ \frac{3}{50} & \frac{91}{100}\end{bmatrix}$$

```
> Eigenvectors(A);
```

$$\begin{bmatrix}\frac{17}{20}\\ 1\end{bmatrix}, \begin{bmatrix}-1 & \frac{3}{2}\\ 1 & 1\end{bmatrix} \qquad (1.1)$$

Why do the eigenvectors form a basis for R^2?

$$x_0 = c_1 \frac{17}{20} \begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_2 1 \begin{pmatrix} \frac{3}{2} \\ 1 \end{pmatrix}$$

using the definition of basis. Also, by applying powers of A to the equation, matrix multiplication turns into scalar multiplication because of the definition of eigenvectors, and so

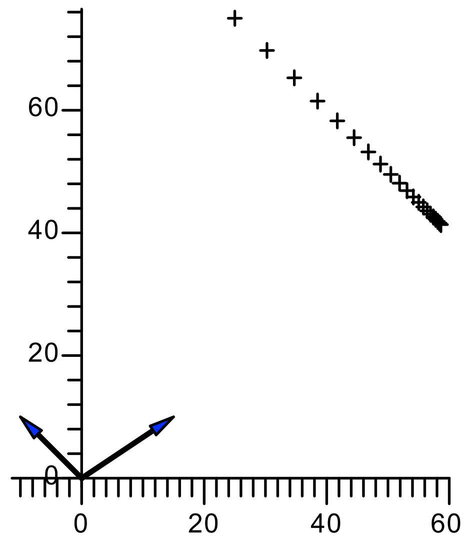
$$x_1 = A x_0 =$$

$$c_1 A \begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_2 A \begin{pmatrix} \frac{3}{2} \\ 1 \end{pmatrix} = c_1 \frac{17}{20} \begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_2 1 \begin{pmatrix} \frac{3}{2} \\ 1 \end{pmatrix}$$

$$x_2 = A x_1 = c_1 \left(\frac{17}{20} \right)^2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_2 (1)^2 \begin{pmatrix} \frac{3}{2} \\ 1 \end{pmatrix}$$

$$x_k = c_1 \left(\frac{17}{20} \right)^k \begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_2 (1)^k \begin{pmatrix} \frac{3}{2} \\ 1 \end{pmatrix}$$

Let's examine what happens in the longerm when the urban population is 25% and the urban is 75%.



$\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ has a smaller effect over time,

until the population ends up fixed on the y $= \frac{2}{3}x$

line. Since the population adds **to** 100%,
this occurs at the x = 60,
y = 40 ratio. Notice that we come **in** parallel **to** y =−
x, since $c_2\,(1)^k \left(\begin{array}{c} \frac{3}{2} \\ 1 \end{array}\right)$ is a constant,
so the only change is **in** $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$